Explaining escalating prices and fines: A unified approach☆

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ABSTRACT

This paper provides an explanation for escalating prices and fines based on a unified analytical framework that nests monopoly pricing and optimal law enforcement. We show that escalation emerges as an optimal outcome if the principal (i) lacks commitment ability, and (ii) gives less than full weight to agent benefits. Escalation is driven by decreasing transfers for non-active agents rather than increasing transfers for active agents. Some forward-looking agents then strategically delay their activity, which drives a wedge between the optimal static transfer and the benefit of an indifferent agent. This wedge is the source of escalation.

1. Introduction

Escalating fines for repeat offenders are ubiquitous, but they pose a serious challenge for the theory of optimal law enforcement. Why should the fine for a given offense increase with the number of previously detected offenses? Escalating pricing schemes for repeat customers (e.g., loyal insurance buyers) pose a similar challenge. Why should loyal customers pay higher prices than new ones? Theory struggles with answering these questions when the economic environment does not change over time.[1]

At first glance, there appears to be a simple explanation for escalation: The principal infers from observed past behavior that active agents have higher unobserved benefits than non-active agents and chooses escalating transfers for previously active agents to extract (part of) these higher benefits. However, this explanation is not correct. In a fixed economic environment it is not optimal to increase the transfers for active agents due to their positive selection (Tirole, 2016). This paper proposes an alternative explanation for escalation that builds on related work in the field of dynamic price discrimination.

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[1] In a recent interview on www.thepolitic.org (August 4, 2018), Avinash Dixit suggests that the formal modelling of graduated punishments is "one of those unresolved research problems."

We develop a unified analytical framework that nests (among other settings) behavior-based monopoly pricing (Armstrong, 2006; Fudenberg and Villas-Boas, 2007) and optimal law enforcement (Becker, 1968; Polinsky and Shavell, 2007) with history-based fines as special cases. Specifically, we allow the objective function of the principal to depend on the weight that the principal gives to agent benefits. The principal thus effectively maximizes monopoly profit if she gives no weight to agent benefits, while she maximizes standard welfare if she gives full weight to agent benefits. In addition, we allow for imperfect detection of agent activity by the principal to accommodate the canonical model of optimal law enforcement. Note that the unified framework assumes that a monetary fine may be viewed as a price (Gneezy and Rustichini, 2000).[2]

The analysis of the unified framework reveals that, contrary to what intuition might suggest, escalation (if any) is driven by decreasing transfers for non-active agents rather than increasing transfers for active agents. The result obtains from the following logic: If the principal (i) cannot commit to future transfers, and (ii) gives less than full weight to agent benefits, she has an incentive to decrease the transfer for non-active agents, thereby generating additional transfer payments from previously non-active agents. Some forward-looking agents will then strategically delay their activity in order to benefit from the lower transfer for non-active agents in the future. This strategic delay drives a wedge between the optimal (expected) static transfer and the (cutoff) benefit of an agent that is indifferent between being active and non-active. This wedge causes the escalation of transfers for active agents. If there is no such wedge, the positive selection of active agents dictates that escalation cannot be optimal.

We develop our line of argument in a simple two-period model. We assume that agent benefits are continuously distributed and fixed over time, and we suppose that the principal and agents share the same discount factor. In period 1, forward-looking agents decide whether or not to engage in the activity, and both active and non-active agents may choose to be active in period 2. The principal detects agent activity with exogenous probability.[3] This implies that, in period 2, the principal can distinguish two groups of agents with different histories: active agents and non-active agents, where the latter were either indeed not active in period 1 ('true' non-active agents) or were active but not detected in period 1 ('false' non-active agents). The principal can set three transfers: The transfer in period 1, the transfer for (true and false) non-active agents in period 2, and the transfer for active agents in period 2.

We derive three key results. First, if the principal can commit to future transfers, it is never optimal to choose escalating transfers for active agents. This finding is reminiscent of the classic result that it is optimal not to discriminate prices with commitment when types are fixed (Stokey, 1979; Hart and Tirole, 1988; Acquisti and Varian, 2005; Fudenberg and Villas-Boas, 2007). Specifically, we show that with commitment the principal can do no better than set all transfers equal to the optimal static transfer. It is worth noting that setting static transfers is not uniquely optimal: falling transfers for active agents may also be optimal. Second, optimal transfers for active agents escalate if and only if the principal lacks commitment ability and gives less than full weight to agent benefits. In this case, the principal cannot resist the temptation to lower the transfer for non-active agents to generate additional transfer payments. Third, if the principal gives full weight to agent benefits, she effectively maximizes standard social welfare and therefore sets all expected transfers equal to the social cost of the activity, irrespective of commitment ability. In sum, escalation is thus explained by the effect of Coasian dynamics (Coase, 1972; Hart and Tirole, 1988).

Our paper makes a twofold contribution. First, we contribute to the literature on behavior-based price discrimination by adding two novel ingredients to the analysis. The first ingredient is imperfect (probabilistic) customer recognition, which allows us to nest the analysis of optimal law enforcement. The paper closest to ours is Conitzer et al. (2012), which studies deterministic customer recognition in a two-period model with repeat purchases. In a recent paper, Belleflamme and Vergote (2016) study imperfect customer identification in a monopoly setting without repeated purchases. Our paper is also related to Villas-Boas (2004), which studies a setting in which an infinitely-lived firm faces overlapping generations of two-period-lived consumers and cannot distinguish 'young' from 'old' first-time consumers. Our analysis differs from much of the customer recognition literature in that we consider a continuous type distribution.[4] The second ingredient is non-profit maximization by the seller. As discussed above, we find that a welfare-maximizing seller does not want to discriminate prices, irrespective of its commitment ability. The reason is that the seller can do no better than set all prices equal to the social cost of consumption. With less weight given to consumer benefits, the seller's profit motive kicks in, and prices are optimally being discriminated.

Second, we contribute to the theory of optimal law enforcement (Polinsky and Shavell, 2007) by providing a novel explanation for escalating fines that builds on history-based fine discrimination in the canonical model. In the spirit of Polinsky and Rubinfeld (1991), we relax the standard assumption that offender gains are fully credited to welfare, which has long been criticized on the grounds that it is difficult to see why "illicit" individual offender gains should add to social welfare (Stigler, 1974; Lewin and Trumbull, 1990). But where Polinsky and Rubinfeld (1991) assume that the "socially-acceptable"

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[2] This is a fairly natural assumption: As will become clear below, the monopoly pricing model and the canonical model of optimal law enforcement are formally equivalent if the principal gives no weight to agent benefits and perfectly detects agent activity.

[3] That is, law enforcement may be uncertain (Polinsky and Shavell, 2007), or consumption may be subject to payment evasion (Buehler et al., 2017). Examples for payment evasion include digital piracy, shoplifting, fare dodging, etc.

[4] Discrete types may provide another rationale for escalation that is driven by the ability to separate types in the second period (e.g. Acquisti and Varian, 2005; Taylor, 2004). With a continuous type distribution instead, there is no incentive to ratchet up (Freixas et al., 1985) the price for revealed high types, and escalation is driven by individuals that strategically delay their offense.

gains of individuals are stochastic, while their "illicit" gains are fixed, we instead allow the principal to place a weight on offender gains in the welfare function. Our analysis shows that the standard assumption of giving full credit to offender gains prevents the canonical model from addressing escalation, as standard welfare maximization forces expected fines down to the social cost of an offense. By allowing for the partial credit of offender gains, we examine the role of a "rent-seeking" principal (Garoupa and Klerman, 2002) in a dynamic framework. Our analysis suggests that dynamic price discrimination offers a consistent explanation for escalating fines in various settings. It should be clear, though, that the economic approach to explaining law enforcement practices has its limits: If the principal has commitment ability or maximizes standard welfare, history-based fine discrimination cannot explain escalating fines.

Earlier work on optimal law enforcement has suggested alternative explanations to solve the 'puzzle' of escalating fines (see Hylton (2005) and Miceli (2013) for useful surveys). For example, law enforcement may be error-prone, so accidental and real offenders are more distinguishable when the number of offenses increases (Stigler, 1974; Rubinstein, 1979; Chu et al., 2000; Emons, 2007). Similarly, if repeat offenders learn how to avoid detection, escalating fines may keep notorious offenders deterred (Baik and Kim, 2001; Posner, 2007).[5] Moreover, if conviction carries a negative social stigma, escalating fines may be needed to keep up deterrence for previously convicted offenders (Rasmusen, 1996; Funk, 2004; Miceli and Bucci, 2005). Also, if the authority minimizes the sum of harm from offenses and the cost of penalization, escalation may be optimal if the cost of penalization is increasing in the level of fines (Endres and Rundshagen, 2016). Finally, escalating fines may be optimal if monitoring is imperfect in the sense that some agents are falsely identified as active, administering punishment is costly, and agents have different types (van der Made, 2019). None of these explanations is based on dynamic price discrimination.

The remainder of the paper is organized as follows. Section 2 introduces the static version of the analytical framework and derives the optimal transfer. Section 3 studies optimal transfers in the two-period version of the model, both with and without commitment by the principal. Section 4 illustrates the analysis with three examples. Section 5 discusses how the results are affected by changes in the unified setting. Section 6 offers conclusions and directions for future research.

2. Static model

We introduce a unified analytical framework that nests monopoly pricing and optimal law enforcement (Becker, 1968; Polinsky and Shavell, 2007) as special cases. Consider a population of agents who obtain the benefit $b \geq 0$ from engaging in an activity (i.e., purchasing a product or committing an offense) that generates (social or private) cost $c \geq 0$. Individual agent benefits are private knowledge and drawn independently from a distribution with density function $z(b)$ and cumulative distribution function $Z(b)$ on $[\underline{b}, \bar{b}]$, with $\bar{b} > c \geq \underline{b}$ and $z(b) > 0$ for all b, such that it is inefficient if no agent engages in the activity. An agent's activity is detected by the principal (i.e., the seller or the law enforcement authority) with probability $\pi \in (0, 1]$, in which case the agent must pay the transfer $t \geq 0$ to the principal. Agents are risk-neutral, implying that only agents whose benefit exceeds the expected transfer, $b \geq \pi t$, engage in the activity.

The principal's objective function is given by

$$\Omega(t; c, \pi, \alpha) = \int_{\pi t}^{\bar{b}} (\pi t - c)dZ(b) + \alpha \int_{\pi t}^{\bar{b}} (b - \pi t)dZ(b), \tag{1}$$

where the first term is the sum of expected transfer payments net of cost, and the second term is the sum of agent benefits net of expected transfer payments, weighted by the parameter $\alpha \in [0, 1]$. It is straightforward to see how monopoly pricing and optimal law enforcement are nested into this framework. First, if $\alpha = 0$ the principal gives no weight to agent benefits and the objective function simplifies to $\Omega(t; c, \pi, 0) = \int_{\pi t}^{\bar{b}} (\pi t - c)dZ(b)$, which is equivalent to the profit function of a monopolist with unit cost c that sells at price πt to a population of consumers with unit demand. Second, if $\alpha = 1$ the principal gives full weight to agent benefits and the objective function simplifies to $\Omega(t; c, \pi, 1) = \int_{\pi t}^{\bar{b}} (b - c)dZ(b)$, which is equivalent to the standard welfare function considered in the canonical model of optimal law enforcement. Third, if $\alpha < 1$, the principal gives less than full weight to agent benefits and has an incentive to extract (some of) these benefits via transfer payments. In the pricing interpretation of the model, it is convenient to think of the principal as a (state-owned) monopoly that does not focus on pure profit only. In the law enforcement interpretation, it makes sense to think of the principal as a law enforcement authority that does not fully credit offender gains to social welfare. Our framework thus relaxes the standard assumption that illicit offenders gains are fully credited to welfare, which has long been criticized in the literature on optimal law enforcement (Stigler, 1974; Lewin and Trumbull, 1990; Polinsky and Shavell, 2007).

Our first result characterizes the optimal static transfer.

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[5] Some authors have argued, though, that declining penalty schemes are optimal if law enforcement becomes more effective in pursuing notorious offenders (e.g. Dana, 2001, Mungan, 2009). Similarly, wealth constraints may make decreasing fines optimal (e.g. Anderson et al., 2017), or lead to falling fines for first offenses over time, but constant ones for repeat offenses (Polinsky and Shavell, 1998).
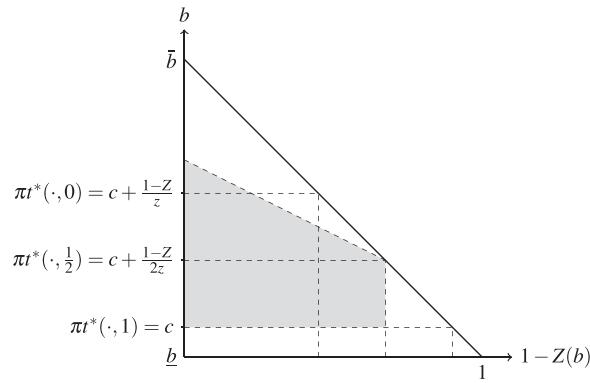
Fig. 1. Static model. *Notes:* The figure illustrates the optimal expected static transfer $\pi t^*(\cdot, \alpha)$ in the unified framework with a uniform distribution of agent benefits and $\alpha \in \{0, 1/2, 1\}$. The shaded area indicates the principal's surplus Ω for $\alpha = 1/2$.

Proposition 1 (static transfer). *Suppose the objective function $\Omega(t; h, \pi, \alpha)$ has a unique interior maximum for any $\alpha \in [0, 1]$. Then, the optimal static transfer satisfies*

$$t^*(c, \pi, \alpha) = \frac{c}{\pi} + \frac{(1-\alpha)[1 - Z(\pi t^*)]}{z(\pi t^*)\pi},$$

with $dt^*(c, \pi, \alpha)/d\alpha \leq 0$.

Proposition 1 shows that the optimal static transfer depends on the weight that the principal gives to agent benefits. If the principal gives no weight to agent benefits ($\alpha = 0$), the optimal transfer takes the form of a standard monopoly price (adjusted for the detection probability). If the principal gives less than full weight to agent benefits ($\alpha < 1$), the optimal transfer is smaller than the standard monopoly price, but larger than the welfare-maximizing transfer c/π, which emerges if the principal gives full weight to agent gains ($\alpha = 1$). In the latter case, the transfer's only role is to discourage agents with a benefit below cost (i.e., consumers with valuation below cost or "inefficient" offenders) from engaging in the activity. The result suggests that it is quite natural to view a fine as a price (Gneezy and Rustichini, 2000): the optimal fine is formally equivalent to the monopoly price if the principal gives no weight to agent benefits and perfectly detects agent activity. Fig. 1 illustrates the unified framework with a uniform distribution of agent benefits and three different values for α. The shaded area corresponds to the principal's surplus if $\alpha = \frac{1}{2}$.

3. Dynamic model

Consider now a repeated version of the unified framework with two periods. Suppose that the principal and agents have the same discount factor $\delta \in (0, 1)$, and assume that the principal can choose three transfers $\mathbf{t} = \{t_1, t_2, \hat{t}_2\}$ to be paid by active agents: t_1 in period 1, t_2 in period 2 for previously non-active agents (both true and false), and $\hat{t}_2$ in period 2 for previously active agents. Note that the principal can condition the transfers in period 2 on the detected agent activity in period 1. Finally, assume that agents are forward-looking and cannot commit to future actions.

Since agents with higher types have higher benefits from being active, the skimming property (Fudenberg et al., 1985; Cabral et al., 1999; Tirole, 2016) ensures that higher-type agents become active no later than lower-type agents. Specifically, if a type b is active in a given period, then so is a higher type $b' > b$. To see how the skimming property works in our setting, observe that for type b to be active in period 1 ($x_1 = 1$), the benefit from being active in period 1 plus the continuation valuation in period 2 must exceed the continuation valuation in period 2 following a decision to be non-active in period 1 ($x_1 = 0$),

$$b - \pi t_1 + \delta V(b, x_1 = 1) \geq \delta V(b, x_1 = 0),$$

where $V(b, x_1)$ denotes the continuation valuation conditional on type b and the activity decision $x_1 \in \{0, 1\}$ in period 1. Since type b can always mimic type $b' > b$ in period 2 (irrespective of activity decisions in period 1), we must have

$$b' - b \geq V(b', x_1) - V(b, x_1), \quad x_1 \in \{0, 1\}, \tag{2}$$

which implies that there exists a unique cutoff $b_1^*(\mathbf{t})$ that splits the type set into active and non-active agents in period 1. Similarly, in period 2 we have that $b' - \pi t_2 > b - \pi t_2$ and $b' - \pi \hat{t}_2 > b - \pi \hat{t}_2$, so that in each period and each segment there exists a unique cutoff.

We now proceed to characterizing optimal agent behavior for any combination of transfers that the principal may choose.
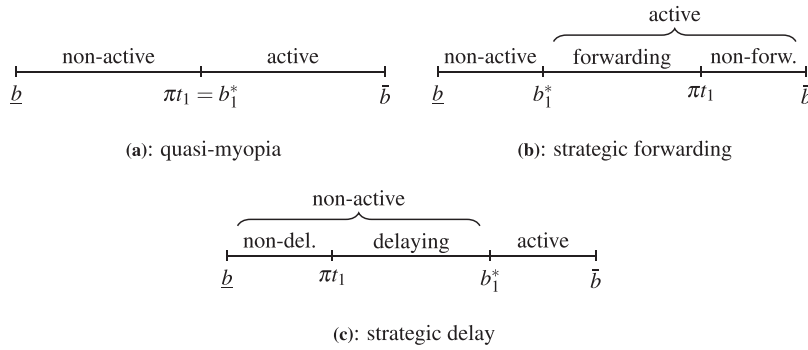
active
┌────────────┐
non-active　　　active　　　　　non-active　forwarding　non-forw.

$\underline{b}$　　　$\pi t_1 = b_1^*$　　$\bar{b}$　　$\underline{b}$　　b_1^*　　　πt_1　　$\bar{b}$

(a): quasi-myopia　　　　　　(b): strategic forwarding

non-active
┌────────────────┐
non-del.　delaying　　active

$\underline{b}$　πt_1　　　b_1^*　$\bar{b}$

(c): strategic delay

Fig. 2. Self-selection in period 1. *Notes:* The figure illustrates how agents optimally self-select in period 1 according to parts (ii)–(iv) of Proposition 2. Panel (a) shows the case of weakly increasing transfers. Panel (b) shows the case of decreasing transfers for active agents. Panel (c) shows the case of decreasing transfers for non-active agents.

Proposition 2 (self-selection). *Forward-looking agents optimally condition their activity on types as follows:*

   (i) *Types $b < \pi \min\{\mathbf{t}\}$ are never active.*
  (ii) *The cutoff satisfies $b_1^* = \pi t_1$ ("quasi-myopia") if $t_1 \leq \min\{t_2, \hat{t}_2\}$ or $t_2 = \hat{t}_2$. Then, types $b \geq \pi t_1$ are active in the first period and active again in the second if they were not detected and $b \geq \pi t_2$, or if they were detected and $b \geq \pi \hat{t}_2$.*
 (iii) *The cutoff satisfies $b_1^* \leq \pi t_1$ ("strategic forwarding") if $t_1 > \hat{t}_2$ and $t_2 > \hat{t}_2$. Then, types $b \in [b_1^*, \pi t_1]$ are active in the first period despite incurring a loss and active again in the second if they were detected, or if they were not detected and $b \geq \pi t_2$.*
 (iv) *The cutoff satisfies $b_1^* > \pi t_1$ ("strategic delay") if $t_1 > t_2$ and $\hat{t}_2 > t_2$. Then, types $b \in [\pi t_1, b_1^*]$ delay their activity despite foregoing a gain in the first period and are active in the second period, and types $b \geq g_1^*$ are active in the first period and active again in the second if $b \geq \pi \hat{t}_2$.*

Proposition 2 characterizes how forward-looking agents optimally condition their activity on their types for any possible combination of transfers. Essentially, three cases (corresponding to parts (ii)–(iv) of Proposition 2) need to be distinguished.

First, if both second-period transfers are weakly higher than the transfer in the first period, forward-looking agents behave as if they were myopic and the cutoff is equal to the myopic level, $b_1^* = \pi t_1$ ("quasi-myopia"). That is, in either period agents are active only if their instantaneous net benefit is weakly positive.[6] Intuitively, agents cannot gain from strategic forwarding if the transfer for previously active agents in the second period exceeds the transfer for active agents in the first period. Similarly, agents cannot benefit from strategic delay because there is no possibility of making up for the foregone benefit in the second period if the transfer for first-time activity increases. Strategic behavior is also excluded if the second-period transfers for previously non-active and active agents are the same, since the surplus that can be obtained in the second period then does not depend on first-period behavior and hence the activity decision in the second period is irrelevant for the optimal first-period decision. In addition, since the second period is the final period of the game, all agents behave myopically when facing second-period transfers. This case is illustrated in panel (a) in Fig. 2.

Second, if the second-period transfer for repeated activity is lower than the first-period transfer, some agents may benefit from strategically being active in the first period to self-select into the set of agents who face the transfer for repeated activity in the second period. However, this will only occur if the transfer for repeated activity is lower than the second-period transfer for first-time activity. The cutoff is then below the myopic level, $b_1^* < \pi t_1$ ("strategic forwarding"). This case is illustrated in panel (b) in Fig. 2.

Third, if the transfer for first-time activity is falling over time, some agents have an incentive to strategically delay their activity. However, this will only occur if the second-period transfer for previously non-active agents is lower than the second-period transfer for repeatedly active agents. In this case, the cutoff exceeds the myopic level, $b_1^* > \pi t_1$ ("strategic delay"), as illustrated in panel (c) in Fig. 2.

Next, we study how the principal optimally chooses the menu of transfers $\mathbf{t}$, accounting for self-selection by agents. In doing so, the principal may or may not be able to commit to the menu of transfers at the beginning of period 1. We consider each case in turn.

3.1. Commitment

Suppose that the principal is able to commit to the full menu of transfers $\mathbf{t} = \{t_1, t_2, \hat{t}_2\}$ at the beginning of period 1. The principal then maximizes the following objective function

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[6] Myopic individual behavior, which refers to behavior that is not forward-looking, is sometimes also called 'naivety' in the literature (e.g. in Taylor, 2004).

$$\Omega = \Omega_1 + \delta(\hat{\Omega}_2 + \Omega_2)$$

$$= \int_{b_1^*}^{\bar{b}} (\pi t_1 - c)dZ(b) + \alpha \int_{b_1^*}^{\bar{b}} (b - \pi t_1)dZ(b) \tag{3}$$

$$+ \delta\pi \left[\int_{\max\{b_1^*, \pi \hat{t}_2\}}^{\bar{b}} (\pi \hat{t}_2 - c)dZ(b) + \alpha \int_{\max\{b_1^*, \pi \hat{t}_2\}}^{\bar{b}} (b - \pi \hat{t}_2)dZ(b) \right] \tag{4}$$

$$+ \delta \left[\int_{\min\{b_1^*, \pi t_2\}}^{b_1^*} (\pi t_2 - c)dZ(b) + \alpha \int_{\min\{b_1^*, \pi t_2\}}^{b_1^*} (b - \pi t_2)dZ(b) \right] \tag{5}$$

$$+ \delta(1 - \pi) \left[\int_{\max\{b_1^*, \pi t_2\}}^{\bar{b}} (\pi t_2 - c)dZ(b) + \alpha \int_{\max\{b_1^*, \pi t_2\}}^{\bar{b}} (b - \pi t_2)dZ(b) \right], \tag{6}$$

where (3) is the surplus generated in period 1, (4) is the discounted second-period surplus from repeatedly active agents, (5) is the discounted second-period surplus from true non-active agents, and (6) is the discounted second-period surplus from false non-active agents. Note that, depending on the relative sizes of the transfers that the principal commits to, the objective function allows for four different cases, which differ with respect to the lower bounds of the respective integrals.

The next result establishes that under commitment it is optimal not to vary the transfers.

Proposition 3 (commitment)**.** *Suppose the principal can commit to the full menu of transfers at the beginning of period 1. Then, she can do no better than set all transfers equal to the optimal static transfer, that is, $t_1^* = t_2^* = \hat{t}_2^* = t^*(c, \pi, \alpha)$.*

The result shows that the principal can do no better than achieve the optimal static outcome in both periods: With commitment, it is optimal to set all transfers equal to the optimal static transfer t^*, irrespective of agents' activity.[7] In the classic monopoly pricing interpretation, Proposition 3 implies that it is optimal to charge the profit-maximizing static monopoly price $t^*(c, 1, 0) = c + \frac{1 - Z(t^*)}{z(t^*)}$ to all consumers, which is in line with earlier work on behavior-based price discrimination by Armstrong (2006) and Fudenberg and Villas-Boas (2007). In the canonical law enforcement interpretation, in turn, the result implies that it is best to impose the static welfare-maximizing fine $t^*(c, \pi, 1) = \frac{c}{\pi}$ on all detected offenders. It is worth noting that constant transfers are not uniquely optimal. Decreasing transfers for agents that are active in both periods may also be optimal if they implement equal cutoffs $b_1^* = b_2^*$. In this case, only agents that are detected twice benefit from the lower transfer in period 2, whereas previously non-detected agents pay the optimal static transfer in period 2. This corresponds to case (iii) in Proposition 2.

Proposition 3 clarifies why the literature has struggled with explaining escalating prices and fines. As long as the principal has commitment ability, the canonical models in the respective strands of literature simply cannot generate escalation as an optimal outcome. We next consider how the lack of commitment affects the principal's incentive to discriminate transfers.

3.2. Non-commitment

Consider a setting in which the principal lacks commitment ability. Optimal transfers in period 2 will then account for (i) the *right*-truncation of the set of previously non-active agents, and (ii) the *left*-truncation of the set of previously active agents, as the cutoff in period 1, b_1^*, separates the type set into non-active $[\underline{b}, b_1^*]$ and active agents $[b_1^*, \bar{b}]$, respectively. The following result shows the implications for the optimal setting of transfers.

Lemma 1 (truncation)**.** *Suppose that the principal lacks commitment ability. Then, she optimally sets the transfers such that strategic delay is the only way in which agents may benefit from strategic behavior, and the cutoff satisfies $b_1^* \geq \pi t_1$.*

The intuition behind Lemma 1 is as follows. Since the principal has no incentive to leave any rent to the lowest type in the set of repeatedly active agents, and there are additional benefits to be extracted from the set of previously non-active agents, it must be that $\hat{t}_2^* \geq t_2^*$. Given these second-period transfers, Proposition 2 shows that agents will either behave as if they were myopic (as in case (ii)) or strategically delay activity (as in case (iv)), depending on the transfers chosen. Note that right-truncation at b_1^* does not eliminate all types $b \geq b_1^*$ from the pool of previously non-active agents in period 2. The reason is that a share $(1 - \pi)$ of the agents with types $b \geq b_1^*$ who are active in period 1 go undetected and thus end up in the pool of previously non-active agents. We now proceed to characterizing optimal transfers in period 2.

3.2.1. Optimal transfers in period 2

We first consider the optimal second-period transfer for repeatedly active agents, $\hat{t}_2^*$. This transfer must maximize the principal's surplus generated from repeatedly active agents with types $b \in [b_1^*, \bar{b}]$,

$$\hat{t}_2^* = \arg\max_{\hat{t}_2 \in \hat{T}_2} \left\{ (\pi \hat{t}_2 - c) \frac{1 - Z(\pi \hat{t}_2)}{1 - Z(b_1^*)} + \alpha(b - \pi \hat{t}_2) \frac{1 - Z(\pi \hat{t}_2)}{1 - Z(b_1^*)} \right\}, \tag{7}$$

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[7] The result is reminiscent of the classic finding that it is optimal not to price discriminate under commitment if consumer types are fixed and all decision makers have the same discount factor (Stokey 1979, Hart and Tirole 1988, Acquisti and Varian 2005, Fudenberg and Villas-Boas 2007).

where $\hat{T}_2 \equiv \{\hat{t}_2 : \pi\hat{t}_2 \geq b_1^*\}$ is the set of transfers for which the expected transfer for repeatedly active agents (weakly) exceeds the cutoff b_1^*. Our next result shows how the optimal transfer is determined.

Proposition 4 (repeated activity). *Suppose that the principal lacks commitment ability. Then,*

(i) *if $b_1^* < \pi t^*(c, \pi, \alpha)$, the optimal second-period transfer for repeatedly active agents equals the optimal static transfer, $\hat{t}_2^* = t^*(c, \pi, \alpha)$.*

(ii) *if $b_1^* \geq \pi t^*(c, \pi, \alpha)$, the optimal second-period transfer for repeatedly active agents keeps the cutoff constant, $\pi\hat{t}_2^* = \hat{b}_2^* = b_1^*$.*

Part (i) states that the optimal second-period transfer for repeatedly active agents equals the optimal static transfer if the cutoff in period 1 is below the optimal static cutoff. The intuition for this result is straightforward: since agents are myopic in period 2 and the left-truncation at b_1^* does not prevent the principal from reaching the static optimum, it is best to choose the optimal static transfer. This finding might suggest that escalation occurs if the initial cutoff is lower than the static optimum. However, it cannot be optimal for the authority to induce a cutoff b_1^* below the static optimum, since this would induce a loss that cannot be recouped in period 2. Henceforth, we therefore focus on the case where b_1^* exceeds the optimal static cutoff.[8]

Part (ii) of Proposition 4 demonstrates that if b_1^* exceeds the optimal static cutoff, the optimal second-period cutoff for repeatedly active agents must be equal to the cutoff from period 1, $\hat{b}_2^* = b_1^*$. That is, the optimal second-period transfer for repeatedly active agents does *not* exclude any previously active agents. This result reflects Tirole (2016)'s insight that the set of inframarginal types is invariant to left-truncation under positive selection. At first glance, the result may seem surprising as cutoff invariance obtains even though non-activity is not absorbing in our setting. Note, however, that the cutoff invariance result holds *only* for repeatedly active agents with types above the cutoff level b_1^* who must have been active in period 1 by construction. Therefore, non-activity is indeed absorbing for repeatedly active agents.[9] Non-activity is clearly not absorbing, though, for agents with types below the cutoff level b_1^*. Importantly, the result implies that the common notion that transfers for repeatedly active agents should be escalating because of their (revealed) higher types is *not* correct. In a fixed economic environment with a continuous type distribution, the principal cannot gain from excluding previously active agents in period 2.

Next, we determine the optimal period-2 transfer for previously non-active agents, t_2^*. This transfer maximizes the principal's surplus generated from true non-active agents with types $b \in [\pi t_2, b_1^*]$ and false non-active agents with types $b \in [b_1^*, \bar{b}]$ that were active in period 1 but went undetected,

$$\Omega_2(t_2; b_1^*, \pi, \alpha) = \int_{\pi t_2}^{b_1^*} (\pi t_2 - c)dZ(b) + \alpha \int_{\pi t_2}^{b_1^*} (b - \pi t_2)dZ(b)$$
$$+ (1 - \pi) \left[\int_{b_1^*}^{\bar{b}} (\pi t_2 - c)dZ(b) + \alpha \int_{b_1^*}^{\bar{b}} (b - \pi t_2)dZ(b) \right]. \tag{8}$$

The next result shows that the optimal transfer for previously non-active agents in period 2 is lower than the optimal static transfer if the principal gives less than full weight to agent benefits.

Proposition 5 (one-time activity). *Suppose that the principal lacks commitment ability. Then,*

(i) *if the principal maximizes welfare, $\alpha = 1$, the optimal second-period transfer for previously non-active agents keeps the cutoff constant, $\pi t_2^* = b_2^* = b_1^* = c$.*

(ii) *if the principal gives less than full weight to agent benefits, $\alpha < 1$, the optimal second-period transfer for previously non-active agents satisfies $\pi t_2^* < b_1^*$.*

Two comments are in order. First, if the principal gives full weight to agents benefits ($\alpha = 1$), the optimal transfer for previously non-active agents in period 2 equals the standard welfare-maximizing transfer, $t_2^* = c/\pi$. This result follows since standard welfare maximization forces the expected transfer down to the cost of the activity. Second, if the principal gives less than full weight to agents benefits ($\alpha < 1$), the optimal expected transfer for previously non-active agents in period 2 is lower than the first-period cutoff. The intuition for this result is straightforward: for any first-period cutoff above the cost of the activity, the principal can gain from lowering the transfer, thereby generating additional transfer payments.

3.2.2. Establishing escalation

We now establish the conditions under which escalation emerges endogenously in the unified framework. Our key result follows immediately from combining the insights from Propositions 2–5.

Proposition 6 (escalation). *Optimal transfers for repeatedly active agents escalate if and only if the principal lacks commitment ability and gives less than full weight $\alpha < 1$ to agent benefits. Optimal transfers for previously non-active agents then fall over*

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[8] Mueller and Schmitz (2015) analyze an offender model in which the initial fines for first-time offenders are exogenously restricted.

[9] Put differently, agents cannot self-select into the set of repeatedly active agents *after non-activity* in period 1.
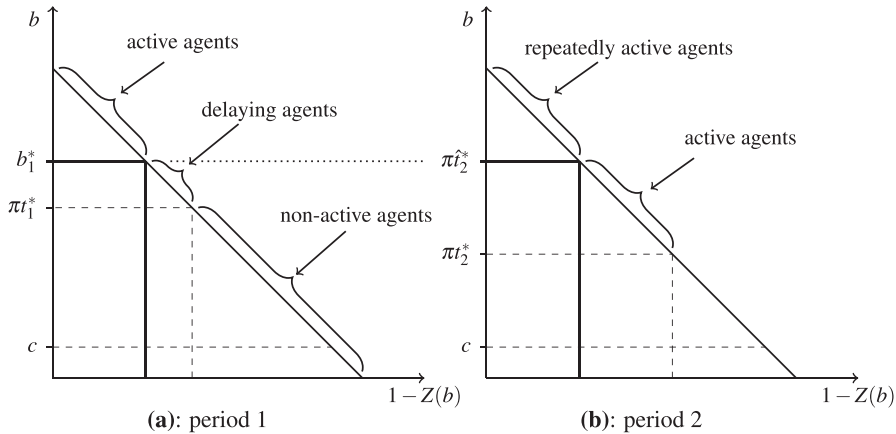
Fig. 3. Dynamic model without commitment. *Notes:* The figure illustrates the optimal transfers and induced inter-temporal cutoff when the principal lacks commitment ability. Panel (a) depicts the first period and shows the wedge between cutoff and expected transfer that delaying agents cause. Panel (b) depicts the second period and shows the resulting escalation in transfers for repeatedly active agents.

time, and transfers are chosen such that

$$\hat{t}_2^* > t_1^* > t_2^*. \tag{9}$$

The result clarifies that two conditions need to be satisfied for escalating transfers to be optimal. First, the principal must lack commitment ability, which prevents her from committing to constant transfers that would yield the highest possible surplus. Second, the principal must give less than full weight to agent benefits, such that optimal transfers do not simply maximize standard welfare in each period.

Proposition 6 highlights that escalating transfers for repeatedly active agents (if any) follow from the principal's incentive to *lower* the transfer for previously non-active agents. The prospect of a decreasing transfer induces some agents to strategically delay their activity, which in turn drives a wedge between the expected transfer πt_1^* and the cutoff b_1^* in period 1. This is illustrated in panel (a) of Fig. 3. The wedge that these delaying agents cause gives rise to escalation, $\hat{t}_2^* > t_1^*$, because by Proposition 4 the cutoff is invariant from period 1 to period 2, $b_1^* = \pi \hat{t}_2^*$, which is illustrated in panel (b) of Fig. 3. In contrast, if there is no wedge between the expected transfer and the cutoff, $\pi t_1^* = b_1^*$, cutoff invariance yields constant transfers $\pi t_1^* = \pi \hat{t}_2^*$.

The intuition for this result is as follows: if the principal gives full weight to agent benefits, transfer payments are irrelevant for the principal's surplus, and optimal expected transfers simply reflect the (constant) cost of the activity. There is thus no incentive to lower the transfer for one-time active agents. However, if less than full weight is given to agent benefits, the benefit extraction motive kicks in and the principal has an incentive to lower the transfer.

4. Examples

We now illustrate how our framework can be applied to derive closed-form solutions for optimal transfers in three different (but inherently related) settings that have been analyzed in the literature. Throughout, we assume that agent benefits are uniformly distributed on [0,1].

4.1. Behavior-based monopoly pricing

Consider the case of behavior-based monopoly pricing as analyzed by Armstrong (2006, pp. 6) and Fudenberg and Villas-Boas (2007, pp. 8). In this setting, the principal gives no weight to agent benefits ($\alpha = 0$), the detection probability is one ($\pi = 1$), and cost is normalized to zero ($c = 0$). The objective function then simplifies to

$$\Pi = \Pi_1 + \delta(\hat{\Pi}_2 + \Pi_2)$$
$$= t_1(1 - b_1^*) + \delta[\hat{t}_2(1 - \max\{b_1^*, \hat{t}_2\}) + t_2(b_1^* - \min\{b_1^*, t_2\})],$$

where the objective function is now denoted by Π instead of Ω. We first consider the case where the monopoly can commit. Applying Propositions 1 and 3, we immediately have that $t_1^* = t_2^* = \hat{t}_2^* = t^* = \frac{1}{2}$.

Next, if the monopoly lacks commitment ability, the price for repeat consumers in period 2 is $\hat{t}_2^* = t^* = \frac{1}{2}$ if $b_1^* < t^*$ and $\hat{t}_2^* = b_1^*$ if $b_1^* \geq t^*$ by Proposition 4. The price for first-time consumers in period 2 must account for right-truncation and is given by $t_2^* = \frac{1}{2} b_1^*$ by Proposition 5. Using these prices, it is straightforward to solve the indifference condition for the cutoff

$b_1^*(t_1) = (2t_1)/(2-\delta)$. Maximizing over t_1 then yields the profit-maximizing prices (Armstrong, 2006)

$$t_1^* = \frac{4 - \delta^2}{2(4+\delta)}; \quad t_2^* = \frac{2+\delta}{2(4+\delta)}; \quad \hat{t}_2^* = \frac{2+\delta}{(4+\delta)}.$$

That is, the monopoly charges escalating prices for repeat consumers ($\hat{t}_2^* > t_1^* > t_2^*$) if it lacks commitment ability, because it cannot resist the temptation to lower the price for first-time consumers in period 2.

This model of behavior-based monopoly pricing can be extended to allow for imperfect detection of consumption ($\pi < 1$). First, imperfect detection leaves both the commitment solution (i.e., constant prices at the static optimal level) and the optimal transfer for repeat consumers in the absence of principal commitment (i.e., expected price at the cutoff) unaffected. Second, in the presence of previously non-active agents in the market in the second period, the monopolist continues to lower the price, yielding escalation. Hence, the mechanics of the analysis are unaffected by the detection probability.

4.2. Monopoly pricing with positive selection

In a recent paper, Tirole (2016) analyzes dynamic monopoly pricing and mechanism design with positive selection, assuming that consumers can consume in future periods only if they have consumed in all previous periods ("absorbing exit"). We consider the basic pricing case where the principal gives no weight to agent benefits ($\alpha = 0$), the detection probability is one ($\pi = 1$), and cost is given by $c > 0$. The objective function then simplifies to

$$\Pi = \Pi_1 + \delta\hat{\Pi}_2$$
$$= (t_1 - c)(1 - b_1^*) + \delta(\hat{t}_2 - c)(1 - \max\{b_1^*, \hat{t}_2\}).$$

With commitment, applying Propositions 1 and 3 immediately yields $t_1^* = t_2^* = \hat{t}_2^* = t^* = \frac{1+c}{2}$. More interestingly, optimal prices are constant even if the monopolist lacks commitment ability. To understand the intuition for this result, note that the assumption of absorbing exit eliminates the surplus from previously non-active consumers in period 2 from the objective function. The monopolist therefore has no incentive to lower the price for these consumers. Instead, she focuses on the surplus from repeat consumers, which is maximized at optimal static prices $t_1^* = \hat{t}_2^* = t^* = b_1^*$. This is in line with the cutoff invariance result of Proposition 4. Hence under positive selection, the ability of the principal to commit is irrelevant (Tirole, 2016).

Note that allowing for imperfect detection of consumption ($\pi < 1$) is inconsequential in the positive selection setting. Since exit is absorbing by assumption, the seller knows that the benefit of any consumer who is still active in period 2 must exceed the first-period cutoff. That is, irrespective of the detected first-period purchase behavior, any buyer in period 2 is a repeat buyer by construction.

4.3. Optimal law enforcement

The canonical model of optimal law enforcement pioneered by Becker (1968) and studied extensively in Polinsky and Shavell (2007) assumes that the principal maximizes standard welfare ($\alpha = 1$) and detects offenses that generate social harm c with probability $\pi \in (0, 1]$. The objective function then simplifies to

$$W = W_1 + \delta(\hat{W}_2 + W_2)$$
$$= (b - c)(1 - b_1^*)$$
$$+ \delta\pi(b - c)(1 - \max\{b_1^*, \pi\hat{f}_2\})$$
$$+ \delta(b - c)(b_1^* - \min\{b_1^*, \pi f_2\})$$
$$+ \delta(1 - \pi)(b - c)(1 - \max\{b_1^*, \pi\hat{f}_2\}),$$

where the objective function is now denoted by W instead of Ω. With commitment, applying Propositions 1 and 3 immediately yields $t_1^* = t_2^* = \hat{t}_2^* = t^* = \frac{c}{\pi}$. More interestingly, the same result holds without commitment, as the principal can do no better than set all expected fines equal to cost, which induces all agents with $b \geq c$ ("efficient offenders") to be active in each period.[10] However, if $\alpha < 1$, the non-discrimination result breaks down, and the principal will optimally set escalating fines if she lacks commitment ability. Note that the mechanism driving these results does not rely on uncertain detection ($\pi < 1$).

5. Discussion

Our analysis shows that escalation is driven by decreasing transfers for non-active agents rather than increasing transfers for active agents in a fixed economic environment. We now discuss how exogenous changes in the environment and heterogenous discount factors affect this finding.

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[10] Note that this result does not depend on the assumption of a uniform distribution. Under welfare maximization ($\alpha = 1$), the optimal static transfer can be determined without knowledge of the distribution.

It should not come as a surprise that exogenous changes in the environment may directly affect the optimal structure of transfers. For instance, if the cost of repeatedly active agents increases from period 1 to period 2, escalation emerges as an optimal outcome even with commitment if $\alpha < 1$. More interestingly, if the cost of previously non-active agents in period 2 is larger than the cutoff in period 1, the principal no longer has an incentive to lower the transfer, which eliminates the commitment problem. A lower detection probability for repeatedly active agents, in turn, induces the principal to increase the transfer for active agents irrespective of commitment if she gives less than full weight to agent benefits. In contrast, if she gives full weight to agent benefits, compensating the fall in expected transfer payments is not uniquely optimal. This follows because transfer payments are welfare-neutral in this case and there is no incentive to exclude previously active agents.

The effects of introducing heterogenous discount factors between the principal and agents are more subtle. With heterogenous discount factors, a given surplus arising in period 2 is valued differently by the principal and agents in period 1. This suggests that it may be beneficial for the principal to shift agent benefits from one period to the other, while keeping their overall benefit constant. That is, when the principal is more patient than agents, $\delta_P > \delta_A$, she has an incentive to *backload* transfers, whereas, if she is less patient, $\delta_P < \delta_A$, she has an incentive to *frontload* transfers. However, the agents' inability to commit prevents the principal from backloading transfers. Nonetheless, escalation will occur even with principal commitment when the agents' discount factor is sufficiently small. In this case, agents essentially behave as if they were myopic, such that there is little (if any) loss for the principal from the strategic behavior of forward-looking agents in period 1. The non-discrimination result from Proposition 3 then collapses, and it becomes optimal for the principal to increase the transfer for active agents and decrease the transfer for non-active agents irrespective of commitment.

6. Conclusion

This paper provided a consistent explanation for escalating prices and fines, using a unified analytical framework that nests monopoly pricing and optimal law enforcement (among other settings) as special cases. Our analysis suggests that escalation emerges as an optimal outcome if and only if the principal (i) lacks commitment ability, and (ii) gives less than full weight to agent benefits.

They key insight of our analysis is that escalation is driven by decreasing transfers for non-active agents rather than increasing transfers for active agents. We suspect that this result is inconsistent with the prima facie intuition of many people (it was at least with ours). It nicely reflects, however, the "curse" of positive selection suggested by Tirole (2016)'s analysis: In a fixed economic environment with a continuous distribution of agent types, it is simply not optimal for the principal to exclude previously active agents.

Our analysis suggests various avenues for future research. First, one could extend the setting to an infinite number of periods. Second, one might examine how competition among sellers affects the scope for escalating pricing schemes. Third, it would be interesting to provide systematic empirical evidence on escalating fines and prices. We hope to address these issues in future research.

Declaration of Competing Interest

None.

Appendix

Proof of Proposition 1. Using Leibniz's rule, differentiating $\Omega(t; c, \pi, \alpha)$ with respect to t yields the first-order condition

$$(1 - \alpha)[(1 - Z(\pi t^*)) - (\pi t^* - c)z(\pi t^*)] = 0.$$

Solving for t^* yields the optimal static transfer $t^*(c, \pi, \alpha)$. The comparative-statics effect of an increase in α on $t^*(c, \pi, \alpha)$ is readily determined by applying the implicit function theorem to the first-order condition and noting that the cross-partial derivative satisfies $\Omega_{t\alpha} = -[1 - Z(\pi t)] \leq 0$. □

Proof of Proposition 2. First, note that the unique cutoff in the first period is determined by the indifference condition $b - \pi t_1 + \delta[\pi(b - \pi \hat{t}_2) + (1 - \pi)(b - \pi t_2)] = \delta(b - \pi t_2)$, where each payoff in the second period is bounded below by zero, as agents may always choose the outside option. We now consider each statement in turn.

(i) Types $b < \pi \min\{\mathbf{t}\}$ make a loss from being active in either period and facing any transfer and hence are never active.

(ii) If $t_1 \leq \min\{t_2, \hat{t}_2\}$, types $b < \pi t_1$ will never be active by (i), while types $b \in (\pi t_1, \pi \min\{\hat{t}_2, t_2\})$ face a loss from being active in period 2 and hence choose the outside option, irrespective of first-period behavior. The indifference condition then simplifies to $b_1^* - \pi t_1 = 0$, which immediately implies $b_1^* = \pi t_1$. Similarly, $t_2 = \hat{t}_2$ implies that the indifference conditions simplifies to $b_1^* = \pi t_1$.

(iii) If $t_1 > \hat{t}_2$, types $b \geq \pi t_1$ face a gain in the second period if they were detected in the first period and face either a loss (and take the outside option) or a gain in the second period if they were not detected. In the first case, the indifference condition simplifies to $b - \pi t_1 + \delta \pi(b - \pi \hat{t}_2) = 0$ which yields $b_1^* < \pi t_1$. In the second case, the indifference condition solves for $b_1^* = \pi(t_1 + \delta \pi(\hat{t}_2 - t_2))$, which yields $b_1^* < \pi t_1$ if $t_2 > \hat{t}_2$.

(iv) If $t_1 > t_2$, types $b \geq \pi t_1$ face a gain in the second period if they were not detected and face either a gain or loss (and take the outside option) in the second period if they were detected. In the first case, the indifference condition solves for $b_1^* = \pi(t_1 + \delta\pi(\hat{t}_2 - t_2))$, which yields $b_1^* > \pi t_1$ when $\hat{t}_2 > t_2$. In the second case, the indifference condition simplifies to $b - \pi t_1 - \delta\pi(b - \pi t_2) = 0$, which yields $b_1^* > \pi t_1$. $\quad\square$

Proof of Proposition 3. Suppose that $b_1^* \geq \pi t^*$, which holds in equilibrium. Then, it cannot be optimal for the principal to commit to a transfer $\hat{t}_2$ such that $\pi\hat{t}_2 > b_1^*$. Similarly, the principal cannot gain from comitting to some $\pi t_2 > b_1^*$ relative to $\pi t_2 \leq b_1^*$. The objective function thus simplifies to

$$
\Omega = \int_{b_1^*}^{\bar{b}} (\pi t_1 - c)dZ(b) + \alpha \int_{b_1^*}^{\bar{b}} (b - \pi t_1)dZ(b)
$$
$$
+ \delta\pi \left[\int_{b_1^*}^{\bar{b}} (\pi\hat{t}_2 - c)dZ(b) + \alpha \int_{b_1^*}^{\bar{b}} (b - \pi\hat{t}_2)dZ(b) \right]
$$
$$
+ \delta \left[\int_{\pi t_2}^{b_1^*} (\pi t_2 - c)dZ(b) + \alpha \int_{\pi t_2}^{b_1^*} (b - \pi t_2)dZ(b) \right]
$$
$$
+ \delta(1-\pi) \left[\int_{b_1^*}^{\bar{b}} (\pi t_2 - c)dZ(b) + \alpha \int_{b_1^*}^{\bar{b}} (b - \pi t_2)dZ(b) \right].
$$

Rewriting this objective function (by collecting terms and splitting up the period-2 surplus from agents that are active in period 2 only) yields

$$
\Omega = \int_{b_1^*}^{\bar{b}} (\pi t_1 - c + \delta\pi(\pi\hat{t}_2 - c))dZ(b) + \alpha \int_{b_1^*}^{\bar{b}} (b - \pi t_1 + \delta\pi(b - \pi\hat{t}_2))dZ(b)
$$
$$
+ \delta \left[\int_{\pi t_2}^{\bar{b}} (\pi t_2 - c)dZ(c) + \alpha \int_{\pi t_2}^{\bar{b}} (b - \pi t_2)dZ(b) \right]
$$
$$
- \delta \left[\int_{b_1^*}^{\bar{b}} (\pi t_2 - c)dZ(b) + \alpha \int_{b_1^*}^{\bar{b}} (b - \pi t_2)dZ(b) \right]
$$
$$
+ \delta(1-\pi) \left[\int_{b_1^*}^{\bar{b}} (\pi t_2 - c)dZ(b) + \alpha \int_{b_1^*}^{\bar{b}} (b - \pi t_2)dZ(b) \right]
$$
$$
= \int_{b_1^*}^{\bar{b}} (\pi t_1 - c + \delta\pi(\pi\hat{t}_2 - c) - \delta\pi(\pi t_2 - c))dZ(b)
$$
$$
+ \alpha \int_{b_1^*}^{\bar{b}} (b - \pi t_1 + \delta\pi(b - \pi\hat{t}_2) - \delta\pi(b - \pi t_2))dZ(b)
$$
$$
+ \delta \left[\int_{\pi t_2}^{\bar{b}} (\pi t_2 - c)dZ(b) + \alpha \int_{\pi t_2}^{\bar{b}} (b - \pi t_2)dZ(b) \right]
$$
$$
= \int_{b_1^*}^{\bar{b}} (b_1^* - c)dZ(b) + \alpha \int_{b_1^*}^{\bar{b}} (b - b_1^*)dZ(b)
$$
$$
+ \delta \left[\int_{\pi t_2}^{\bar{b}} (\pi t_2 - c)dZ(g) + \alpha \int_{\pi t_2}^{\bar{b}} (b - \pi t_2)dZ(b) \right],
$$

where the last step follows from noting that the indifference condition

$$
b_1^* - \pi t_1 + \delta\pi \max\{0, b_1^* - \pi\hat{t}_2\} + \delta(1-\pi)\max\{0, b_1^* - \pi t_2\} = \delta\max\{0, b_1^* - \pi t_2\}
$$

solves for

$$
b_1^* = \pi(t_1 - \delta\pi t_2 + \delta\pi\hat{t}_2). \tag{A.1}
$$

Now we can see that the first term is maximized at $b_1^* = \pi t^*$, while the second term is maximized at $\pi t_2 = \pi t^*$. Using (A.1), it follows that $t_1 = t_2 = \hat{t}_2 = t^*$ maximizes the principal's objective function under commitment and implements $b_1^* = \pi t^* \geq \pi t^*$.

Now suppose that $b_1^* < \pi t^*$. As before the principal cannot gain from committing to some $\pi t_2 > b_1^*$, such that the third and fourth term of the objective function remain unchanged. The second term changes, however, as the lower bound of the integral increases to $\pi\hat{t}_2 > b_1^*$. Analogous calculations then show that the principal obtains a strictly smaller surplus compared to the case where $b_1^* \geq \pi t^*$. $\quad\square$

Proof of Lemma 1. For the left-truncated set of agents that are active in both periods, $[b_1^*, \bar{b}]$, the principal can do no better than leave no rent to the lowest type, hence $\pi \hat{t}_2^* \geq b_1^*$. For the set of previously non-active agents, the principal can do no better than set t_2^* such that $\pi t_2^* \leq b_1^*$. Therefore, we must have that either $\hat{t}_2^* > t_2^*$ or $\hat{t}_2^* = t_2^*$. By Proposition 2, agents then cannot benefit from strategic forwarding and hence $b_1^* \geq \pi t_1$. □

Proof of Proposition 4. We consider both statements in turn.

(i) For $b_1^* < \pi t^*(c, \pi, \alpha)$, it is optimal for the principal to set $\hat{t}_2^* = t^*(c, \pi, \alpha)$ by Proposition 1, as agent behavior is myopic in period 2.

(ii) For $b_1^* \geq \pi t^*(c, \pi, \alpha)$, the surplus in (7) is maximized at the lower bound after left-truncation, $\pi \hat{t}_2^* = \hat{b}_2^* = b_1^*$. □

Proof of Proposition 5. We consider both statements in turn.

(i) For $\alpha = 1$, all agents with $b \geq c$ must be active to maximize welfare. In period 2, this requires that $t_2^* = c/\pi = t^*(c, \pi, \alpha = 1) = \hat{t}_2^*$. Proposition 2 then implies that $b_1^* = \pi t_1^*$, and by Proposition 4 we know that $b_1^* = \pi \hat{t}_2^*$, which implies $t_1^* = c/\pi$.

(ii) For $\alpha < 1$, we must have $b_1^* > c$, as it cannot be optimal to chose transfers that yield $b_1^* = c$. Similarly, $b_1^* < \bar{b}$ holds by construction. The principal can then gain from lowering the expected transfer for previously non-active agents below the cutoff, $c < \pi t_2^* < b_1^*$. □

Proof of Proposition 6. Proposition 3 shows that escalation is not optimal under commitment and thus establishes the necessity of non-commitment. Similarly, Proposition 5 shows that optimal transfers under non-commitment are constant with $\alpha = 1$, which establishes the necessity of $\alpha < 1$. To establish sufficiency, note that Proposition 5 demonstrates that when $\alpha < 1$ and the principal lacks commitment, $t_2^* < b_1^*$, while Proposition 4 shows that $\pi \hat{t}_2^* \in [b_1^*, \bar{b}]$, which immediately implies that $t_2^* < \hat{t}_2^*$. Then by Proposition 2 (some) agents strategically delay their activity, $b_1^* > \pi t_1^*$, and $t_1^* > t_2^*$, which yields the result. □

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