Dynamic Monopoly Pricing With Multiple Varieties: Trading Up*

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Abstract

This paper studies dynamic monopoly pricing for a class of settings that includes multiple durable, multiple rental, or a mix of varieties. We show that the driving force behind pricing dynamics is the seller's incentive to switch consumers—buyers and non-buyers—to higher-valued consumption options by lowering prices ("trading up"). If consumers cannot be traded up from the static optimal allocation, pricing dynamics do not emerge in equilibrium. If consumers can be traded up, pricing dynamics arise until all trading-up opportunities are exhausted. We study the conditions under which pricing dynamics end in finite time and characterize the final prices at which dynamics end.

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1 Introduction

Dynamic monopoly pricing has intrigued economists for decades. The literature has long emphasized that Coasian dynamics are key for understanding dynamic monopoly pricing: The monopolistic seller of a durable good who cannot commit to future prices has an incentive to lower the prices for (negatively selected) non-buyers over time, since high-value buyers purchase early on.[1] Yet, recent work has identified various settings in which monopoly pricing is *not* governed by Coasian dynamics. For instance, there is no commitment problem for the seller, and pricing dynamics do not arise, if the potential buyers of a durable good have access to an outside option with strictly positive value that ends the game (Board and Pycia, 2014). Similarly, optimal prices remain constant if only high-value rather than low-value consumers remain in the market ("positive selection") for a rental good (Tirole, 2016).[2] Finally, if the seller offers two durable varieties (rather than one), then Coasian dynamics apply, but they generally do not lead to zero profits in the limit (Nava and Schiraldi, 2019). Failures of the Coase conjecture have thus been shown to emerge for different reasons in different settings.

This paper studies dynamic monopoly pricing for a broad class of settings that includes previously unexplored settings with multiple rental varieties or "mixed" settings with one durable and one rental variety.[3] The analysis highlights that the driving force behind pricing dynamics—as opposed to the repeated play of static monopoly prices—is the seller's incentive to "trade up" consumers to higher-valued consumption options: Faced with a set of consumers who can be traded up to a higher-valued consumption option, the seller has an incentive to cut the price of this higher-valued option and benefit from the larger surplus emerging after switching consumers. Our notion of trading up extends the logic of Coasian

[1]The lack of commitment constrains the monopolist's market power, and in the limit the profit converges to zero if all trade takes place in the "twinkle of an eye", as conjectured by Coase (1972) and formally established by Stokey (1981), Bulow (1982), Fudenberg et al. (1985), Gul et al. (1986), and Ausubel and Deneckere (1989).

[2]If the seller offers a rental good and both negative selection (for non-buyers) and positive selection (for loyal buyers) are at work, then Coasian dynamics for the prices offered to non-buyers lead to "behavior-based pricing" (Acquisti and Varian, 2005; Armstrong, 2006; Fudenberg and Villas-Boas, 2007; Buehler and Eschenbaum, 2020).

[3]Nava and Schiraldi (2019) study an extension of their setting in which consumers may return to the market after purchase, but focus on the impact on market clearing.

dynamics, which applies to the prices set for non-buyers of a durable good, to the prices set for buyers of a non-durable good.

Specifically, we consider a monopolist with zero marginal cost that chooses prices for two varieties of a good facing a unit mass of consumers with unit demand. We assume that the monopolist cannot commit to future prices. In each period, consumers either consume one of the varieties or refrain from consumption and are thus in one of three states. They all start the game in the same state, and their fixed values of the two varieties are private information. A fixed set of admissible transitions between the three states governs the choices that are available to consumers in every period. Hence, if consumers cannot select one of the varieties throughout the game, our setting reduces to a one-variety problem. For a consumer who may not select the outside option in a given period, we impose a price of zero for her previous consumption choice to prevent expropriation. Thus, an absorbing variety can be viewed as a durable good that can be sold only once, whereas a variety that can be purchased in every period can be viewed as a rental good. We are interested in characterizing the pricing dynamics in Perfect Bayesian Equilibrium (PBE).

We derive three key results. First, we show that, at a history at which there are no trading-up opportunities, the best the seller can do is to let buyers continue with purchasing their most-preferred variety at a constant price for all future periods. Hence, no dynamics in realized consumption choices or paid prices emerge along the equilibrium path starting at such a history. The result is reminiscent of Tirole (2016)'s finding that it is optimal to offer a constant price to loyal buyers of a single rental good in a positive selection setting where the outside option is absorbing.[4] The result highlights that trading-up opportunities are crucial for pricing dynamics to emerge. Second, we show that, if there are no trading-up opportunities in the monopoly outcome of the static game, the seller cannot do better than obtain the repeated static monopoly profit $\pi(p^m)$ over the course of the game. In the essentially unique PBE of the game, the seller obtains the repeated monopoly profit and therefore does not face a commitment problem.[5] The result establishes

[4]In the positive selection setting with a single variety, all consumers begin the game in the state where they consume the variety, and they cannot return to the consumption state after choosing the absorbing outside option. There are thus no trading-up opportunities at any history.

[5]The equilibrium is essentially unique in the sense that the seller obtains the repeated

that the seller cannot benefit from pricing dynamics if she can exhaust all trading-up opportunities by implementing the static monopoly outcome right from the start. The seller can do so, for instance, in a setting where the potential buyers of a durable good have access to an additional durable variety with strictly positive value (similar to the setting considered in Board and Pycia, 2014), or a setting with positive selection with a single variety (Tirole, 2016). Third, we show that for any history at which there are trading-up opportunities, the seller trades up consumers along the equilibrium path following this history. Hence, prices fall until all trading-up opportunities are exhausted. Yet, they do not fall below the prices $\bar{p}$ associated with the seller-optimal outcome in the static game that leaves no trading-up opportunities. In addition, the seller's present discounted profit is bounded from below by the repeated static profit $\pi(\bar{p})$, which implies that the seller can obtain a positive profit in many settings. We further show that whether or not the pricing dynamics are played out in finite time depends on the setting under study and the lowest values in the support.

Our analysis highlights that the pricing dynamics in a broad class of monopoly pricing problems depend on whether or not the monopoly outcome in the static game leaves trading-up opportunities to the seller. If the monopoly outcome leaves no trading-up opportunities, then the seller does not face a commitment problem and the profit-maximizing solution is to implement the repeated static monopoly outcome irrespective of commitment ability. Instead, if there are trading-up opportunities at the static monopoly outcome, then the monopolist will lower prices to trade up consumers over time, and a zero-profit lower bound applies in some settings, but not in general. Our analysis extends the essence of Coase's insight to the pricing of non-durable varieties: pricing dynamics emerge whenever the seller has an incentive to switch consumers to a higher-valued consumption option.

We discuss various implications of our analysis, paying particular attention to settings with two rental or mixed varieties. For instance, the prices of two rental varieties will eventually equalize if the lowest values in the support are the same. Also, the seller of two mixed varieties will be able to obtain a strictly positive profit if there are consumers who prefer the rental to the durable variety. The latter implication extends Nava and Schiraldi (2019)'s insight for two durable varieties—intra-temporal price discrimination can partially make up for the loss of market

monopoly profit in any PBE of the game.

power due to inter-temporal price discrimination and shield the seller from zero profits—to settings in which the seller can exhaust all trading-up opportunities at once while achieving positive profits. Finally, we show how our insights translate to settings in which one variety may only be indirectly accessible to consumers via another state ("transitional games").

This paper contributes to an extensive literature on the pricing of a single durable good (e.g Coase, 1972; Fudenberg et al., 1985; Gul et al., 1986; Sobel, 1991; Kahn, 1986; Bond and Samuelson, 1984; Fuchs and Skrzypacz, 2010), of multidimensional settings with a durable good (e.g. Nava and Schiraldi, 2019; Board and Pycia, 2014), and of vertically differentiated durable products (Hahn, 2006; Inderst, 2008; Takeyama, 2002). Our work differs by proposing a unified analytical framework and focusing on settings with two rentals or mixed varieties, respectively, that have largely gone unnoticed. In doing so, we add to the analysis of positive selection (Tirole, 2016), as we allow for an absorbing outside option. Our framework shows how the analysis of positive selection can be extended to multiple varieties. In addition, we contribute to the literature on behavior-based pricing (e.g. Acquisti and Varian, 2005; Armstrong, 2006; Fudenberg and Villas-Boas, 2007; Taylor, 2004; Buehler and Eschenbaum, 2020). In contrast to recent work by Rochet and Thanassoulis (2019), we focus on settings with unit-demand and do not allow for varieties to be sold as a bundle. Finally, our notion of "trading up" is related to the concept of "upselling" in the marketing literature (e.g., Blattberg et al., 2008; Aydin and Ziya, 2008; Wilkie et al., 1998). The key difference is that upselling refers to the upgrading of loyal buyers to a more expensive product, whereas trading up applies to buyers and non-buyers alike.

The remainder of the paper is organized as follows. Section 2 introduces the analytical framework, formalizes the notion of trading-up opportunities, and sketches various applications. Section 3 provides a skimming property for the unified analytical framework, explains how it translates into known skimming results for specific settings, and characterizes dynamic monopoly pricing in the absence and presence of trading-up opportunities. Section 4 discusses the implications of our results for specific applications. Section 5 examines an extension to transitional games. Section 6 concludes and offers directions for future research.

2 Analytical Framework

Consider a monopolist that offers two varieties of a good, a and b, at zero marginal cost to a unit measure of consumers, and suppose that the monopolist cannot commit to future prices. Consumers have unit-demand and either purchase one of the varieties or select the outside option in each period. Following Nava and Schiraldi (2019), the value profiles of consumers $v = (v_a, v_b)$ are fixed, private information, and distributed according to a measure $\mathcal{F}$ on the unit square $[0,1]^2$. The associated cumulative distribution is F, with density f, and V is the support. F_i denotes the marginal cumulative distribution of variety i, while f_i and V_i denote the respective density and support.[6] The value of the outside option is zero.

Time is discrete and indexed by $t = 0, ..., T$, where T is finite or infinite. All players share the same discount factor $\delta \in (0,1)$. In every period t, buyers make a discrete choice $x^t \in X$, where

$$X \equiv \{(1,0), (0,1), (0,0)\},$$

is the set of states with elements $a = (1,0)$, $b = (0,1)$, and the outside option $o = (0,0)$. Let $\bar{x} \in X$ be the initial state for all consumers. A sequence of choices x^t from period t onward is a *consumption path* $\mathbf{x}^t = (x^t, x^{t+1}, ..., x^T)$ that gives rise to (present discounted) *total consumption* $\chi(\mathbf{x}^t) = \sum_{\tau=t}^{T} \delta^{\tau-t} x^\tau$. A consumption path is *admissible* if all transitions from state to state along the entire path are within the set of admissible transitions $\Gamma \subset X \times X$, where Γ is exogenous and determines how consumers can switch between states from one period to the next. Throughout we maintain the assumption that transitions from a state to itself are always admissible, that is, $(o,o), (a,a), (b,b) \in \Gamma$. In the main part of our analysis we focus on settings in which each state is either directly accessible from the initial state or not accessible at all. We will consider the extension to "transitional games" (where one variety is only indirectly accessible from the initial state) in Section 5. A state $x \in X$ is *absorbing* if no other state $x' \in X$ is accessible from x, that is, $(x, x') \notin \Gamma$. Let $\Delta^t = \sum_{\tau=t}^{T} \delta^{\tau-t}$ denote the (present discounted) *number of periods* from t on.[7]

[6] For instance, in the case of a bivariate uniform distribution of value profiles, the cumulative distribution is $F(c,d) = \int_0^d \int_0^c f(v_a, v_b) dv_a dv_b = cd$, with density $f(v_a, v_b) = 1$, for $0 \le v_a, v_b \le 1$, and the marginal distribution is univariate uniform.

[7] From now on, we will consistently omit the exponent for all expressions if $t = 0$.

A variety that is absorbing and can only be sold once and for all future periods is a durable variety. A variety that allows for transitions to and from the outside option in every period, in turn, is a rental variety. To simplify exposition, we will henceforth refer to the setting with two absorbing varieties and non-absorbing initial state $\bar{x} = o$ as the "two durables" setting. Similarly, we will refer to the setting with all transitions being admissible and initial state $\bar{x} = o$ as the "two rentals" setting. Lastly, we will refer to the setting with one rental and one durable variety with initial state $\bar{x} = o$ as the "mixed varieties" setting.

Figure 1: States and transitions in two nested settings

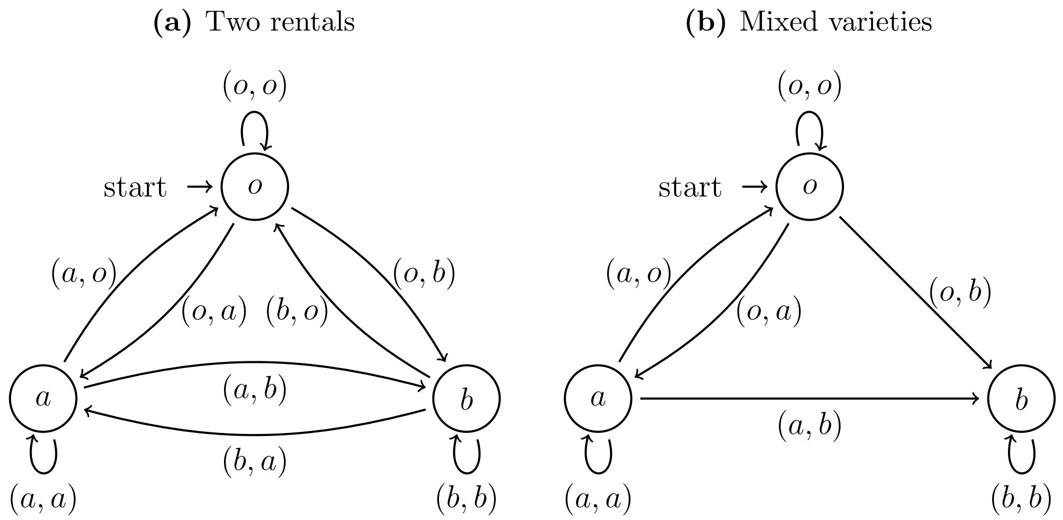


Figure 1 illustrates the two rentals setting and the mixed varieties setting. The vertices indicate the states $X = \{a, b, o\}$, with initial state $\bar{x} = o$, while the arcs and brackets $(x, x') \in \Gamma$ represent the admissible transitions. Panel (a) shows the two rentals setting in which all transitions are admissible, $\Gamma = \{(a, a), (b, b), (o, o), (a, b), (b, a), (o, a), (o, b), (a, o), (b, o)\}$. Panel (b) shows the mixed varieties setting with a as the rental variety and indicates that two transitions are not admissible, $(b, o), (b, a) \notin \Gamma$, because durable varieties are absorbing states.

2.1 Prices, Histories, and Solution Concept

All players are risk-neutral. In each period t, the monopolist selects a price profile $p^t = (p_a^t, p_b^t) \in [\psi, 1]^2$, with $\psi < 0$,[8] for every history of play. Consumers then either purchase one of the varieties or forego consumption. Importantly, if buyers cannot transition from their current state $i \in (a, b)$ to the outside option (i.e., $(i, o) \notin \Gamma$), then the period-t price for variety i, p_i^t, is set to zero by assumption, and the seller only chooses the price for the other variety $j \neq i$. This assumption is consistent with our interpretation of an absorbing variety as a durable good and excludes the expropriation of "captured" buyers. Let $\rho(\mathbf{x}) = \sum_{t=0}^{T} \delta^t (p^t \cdot x^t)$ denote the (present discounted) *total payment* made along consumption path $\mathbf{x} = (x^0, x^1, ..., x^T)$. Similarly, let $\nu(v, \mathbf{x}) = v \cdot \chi(\mathbf{x})$ be the (present discounted) *total value* obtained by a buyer with value profile v along consumption path $\mathbf{x}$. We can then write the (present discounted) total utility obtained by a buyer with value profile v along consumption path $\mathbf{x}$ compactly as $\nu(v, \mathbf{x}) - \rho(\mathbf{x})$.

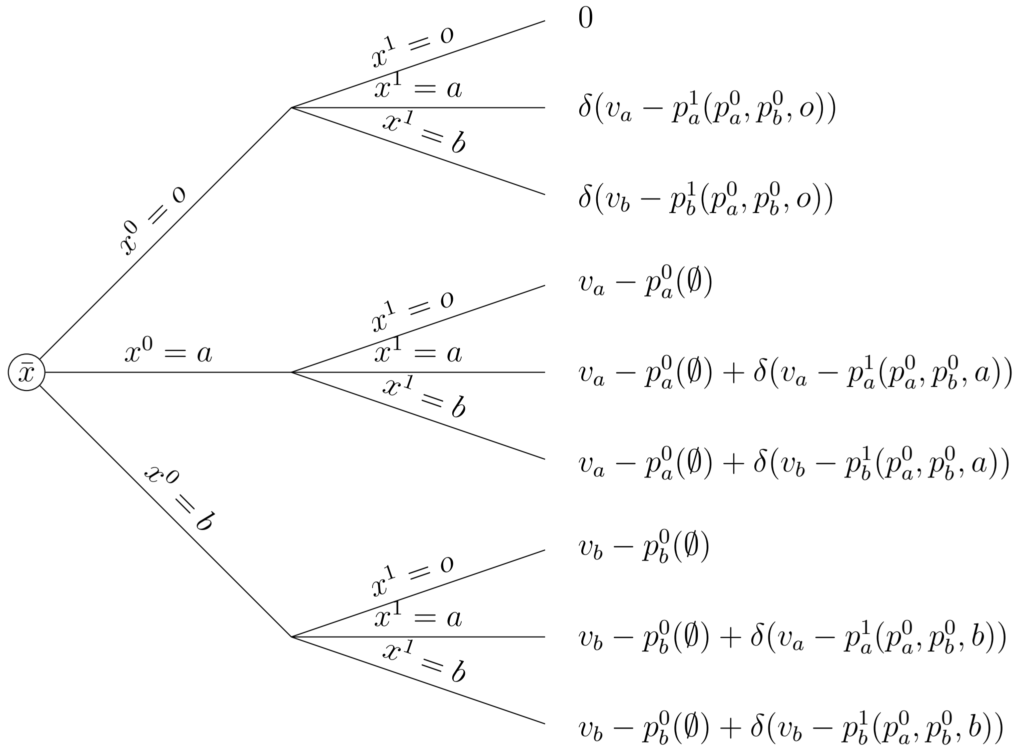
Figure 2 illustrates the admissible consumption paths $\mathbf{x}$ and corresponding utilities obtained by a consumer with value profile $v = (v_a, v_b)$ for two rentals and two periods ($T = 1$). For instance, the lowest branch in Figure 2 depicts the "always-b" path $\mathbf{x}_b = (b, b)$ with total consumption $\chi(\mathbf{x}_b) = (0, 1 + \delta)$, total value $\nu(v, \mathbf{x}_b) = (1 + \delta)v_b$, total payment $\rho(\mathbf{x}_b) = p_b^0 + \delta p_b^1(p_a^0, p_b^0, b)$, and total utility $\nu(v, \mathbf{x}_b) - \rho(\mathbf{x}_b) = v_b - p_b^0(\emptyset) + \delta(v_b - p_b^1(p_a^0, p_b^0, b))$.

A period-t seller history, h^t, is a sequence of previous price profiles $(p^0, ..., p^{t-1})$ and consumption choices $(x^0, ..., x^{t-1})$, with $h^0 = \emptyset$. A period-t buyer history, $\hat{h}^t$, consists of the seller history h^t and the period-t price profile $p^t(h^t) = (p_a^t(h^t), p_b^t(h^t))$ offered to consumers with seller history h^t. The set of period-t seller histories is denoted by H^t, and the set of all seller histories by $H = \cup_{t=0}^{T} H^T$. Similarly, the set of period-t buyer histories is denoted by $\hat{H}^t$, and the set of all buyer histories by $\hat{H} = \cup_{t=0}^{T} \hat{H}^T$. Let $V(h^t) \subseteq V$ denote the subset of consumers with the same seller history h^t.

We let $\Pi(h^t)$ denote the (present discounted) value of the seller's profit in the dynamic game obtained from buyers with history $h^t \in H^t$. The seller's profit in

[8] The assumption on the set of prices ensures that the monopolist's action set is compact.

Figure 2: Consumption paths and utilities for two rentals and two periods



the static game, in turn, is given by

$$\pi(p) = \sum_{i \in (a,b)} p_i \mathcal{F}\left(v \in V \middle| i = \arg \max_{x \in X, (\bar{x}, x) \in \Gamma} \{(v-p)\cdot x\} \right).$$

Let $\pi(p^m)$ denote the supremum of the seller's profit in the static game, henceforth called the "monopoly profit" for convenience, with associated price profile p^m,

A behavioral strategy for buyers is denoted by $\hat{\sigma}$ and determines the probability distributions over the consumption choices $x \in X$ made by buyers at every possible history. In line with the literature, we assume that at any possible history the set of buyers making the same consumption choice is a measurable set. A behavioral strategy for the seller is denoted by σ and determines the probability distribution over the prices $p \in [\psi, 1]^2$ set by the seller as a function of the history of play. A *Perfect Bayesian Equilibrium* (PBE) is a strategy profile $\{\sigma, \hat{\sigma}\}$ and updated beliefs about the buyers' values along the various consumption paths, such that actions are optimal given beliefs, and beliefs are derived from actions from Bayes' rule whenever possible.

2.2 Trading-Up Opportunities

We say that there is a trading-up opportunity for the seller if there are consumers who can transition to a strictly higher-valued state:

Definition 1 (**Trading-up opportunity**). *The seller has a trading-up opportunity if there is a positive measure of consumers in state x for whom transitions to a strictly higher-valued state x' are admissible, that is,*

$$\exists x, x' \in X \ s.t. \ (x, x') \in \Gamma \ and \ v \cdot x' > v \cdot x.$$

Let Ω denote the set of price profiles $p = (p_a, p_b)$ that induce an allocation which leaves no trading-up opportunities for the seller in the static game,

$$\Omega = \left\{ p \in \mathbb{R}^2 \middle| x = \arg \max_{\tilde{x} \in X, (\bar{x}, \tilde{x}) \in \Gamma} (v - p) \cdot \tilde{x} \ \Rightarrow v \cdot x > v \cdot x' \text{ or } (x, x') \notin \Gamma \ \forall v \in V \right\}.$$

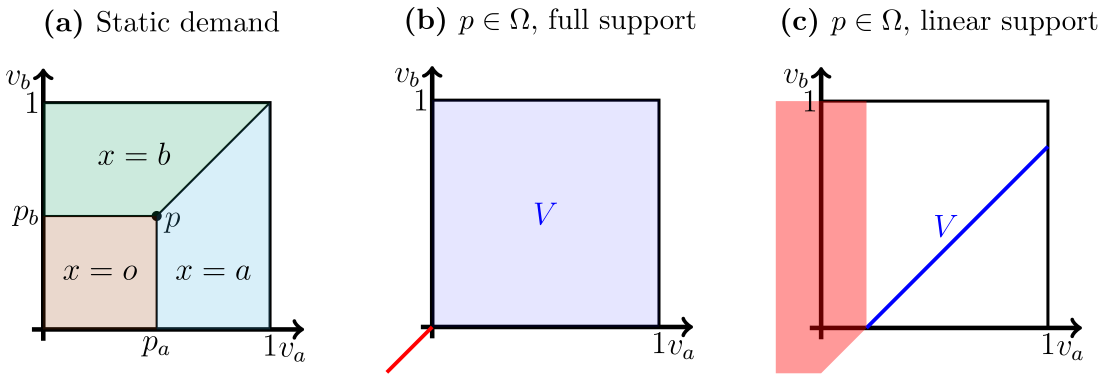
Intuitively, any price profile $p \in \Omega$ must induce an allocation where all consumers either choose their most-preferred state among those that are accessible from the initial state, or an absorbing state. Thus, in a setting with two durables, $p \in \Omega$ is equivalent to market-clearing. In a setting with two rentals, $p \in \Omega$ is equivalent to market-clearing and efficiency.[9] Finally, we let $\bar{p} \in \Omega$ denote a price profile that is associated with the supremum of the profit obtainable in the static game conditional on leaving no trading-up opportunities, $\pi(\bar{p})$. This profit supremum exists provided that $\Omega \neq \emptyset$, which is guaranteed unless there exists a state that is accessible only (indirectly) over the course of the game, but not from the initial state, since otherwise we always have $p = (0, 0) \in \Omega$.[10]

Figure 3 illustrates for the two rentals setting how buyers self-select in the static game for a given price profile p, and depicts price profiles (in red) that satisfy $p \in \Omega$ for two different supports. Specifically, panel (a) shows the static

[9] We follow Nava and Schiraldi (2019) in referring to price profiles which ensure that all buyers choose their most-preferred (accessible) variety as *efficient*, since they maximize total welfare when marginal costs are zero.

[10] We consider the extension to transitional games in section 5.

Figure 3: Demand segments in the static game for given p (panel (a)), and profiles $p \in \Omega$ with full support (panel (b)) or linear support (panel (c)) for two rentals



demand segments for a given price profile $p = (0.5, 0.5)$ and indicates, for instance, that all consumers with a value profile $v < p$ choose the outside option, $x = o$. Panels (b) and (c) depict price profiles that leave no trading-up opportunities with full and linear support, respectively. For two rentals, $p \in \Omega$ requires that all consumers choose their most-preferred variety, as otherwise there are trading-up opportunities from one variety to the other, or from the initial state to each variety. Thus, with full support only non-positive price profiles on the diagonal satisfy $p \in \Omega$ (panel (b)), whereas with an increasing linear support that lies to the right of the diagonal through the type space (panel (c)), any price profile that ensures $x = a$ for all types in the support satisfies $p \in \Omega$, since all buyers prefer a to b ("vertical differentiation").

2.3 Applications

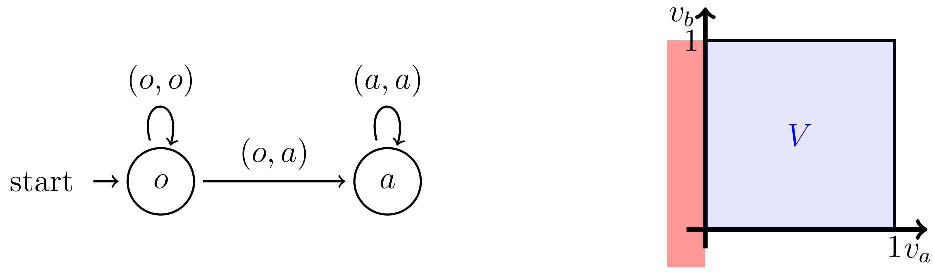
Our analytical framework covers a class of dynamic monopoly pricing settings that are characterized by the tuple $(\bar{x}, \Gamma, \mathcal{F})$ and can be illustrated in two complementary graphs: one showing the accessible states and admissible transitions, and one showing the support V of the value profiles. Figure 4 provides three examples that are drawn for a full support V on the unit square $[0, 1]^2$. We also indicate price profiles that satisfy $p \in \Omega$ in red.[11]

In the setting with a single durable variety a (Figure 4a), $p \in \Omega$ requires that

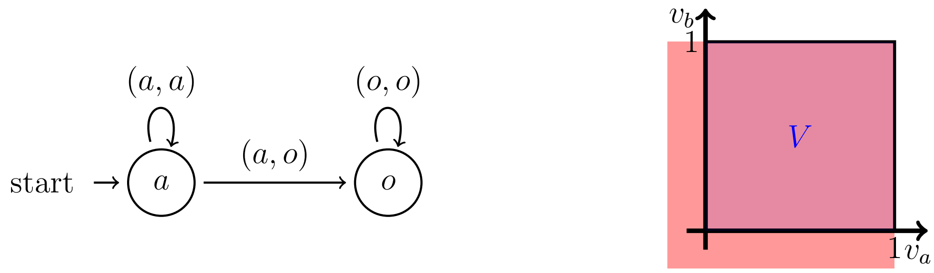
[11] For simplicity, inaccessible states (and transitions out of these states) are omitted.

Figure 4: Three examples: Accessible states and admissible transitions (left), and price profiles $p \in \Omega$ with a full support (right)

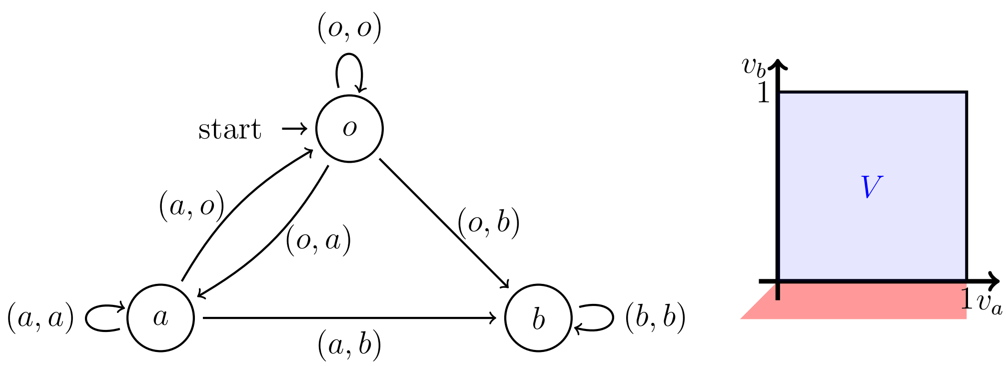
(a) Single durable variety a



(b) Positive selection, with single variety a and initial state $\bar{x} = a$



(c) Mixed setting, with rental variety a, durable variety b, and initial state $\bar{x} = o$



$p_a \leq 0$ (whereas the price p_b remains unrestricted) given a full support, which implies that $\pi(\bar{p}) = 0$. In a setting characterized by positive selection with one variety a and initial state $\bar{x} = a$ (Figure 4b), in turn, all price profiles satisfy $p \in \Omega$, which implies that $\pi(\bar{p}) = \pi(p^m) > 0$ with a full support. Finally, in the mixed setting with rental variety a, durable variety b, and initial state $\bar{x} = o$ (Figure 4c), $p \in \Omega$ requires that the price of the durable variety b is non-positive, whereas the price of the rental variety a can be positive and thus $\pi(\bar{p}) > 0$ with a full support.

We pay particular attention to the price profiles $p \in \Omega$ that leave no trading-up opportunities in the static game because our analysis will demonstrate that, in order to characterize the dynamics in equilibrium, it is sufficient to determine the supremum of the profit obtained in the static game conditional on leaving no trading-up opportunities, $\pi(\bar{p})$, and examine whether it coincides with the monopoly profit $\pi(p^m)$.

3 Analysis

In this section, we employ the unified framework to characterize dynamic monopoly pricing. We proceed in three steps. First, we examine the behavior of consumers in equilibrium. Second, we study optimal pricing in the absence of trading-up opportunities. Finally, we analyze optimal pricing with trading-up opportunities.

3.1 Skimming

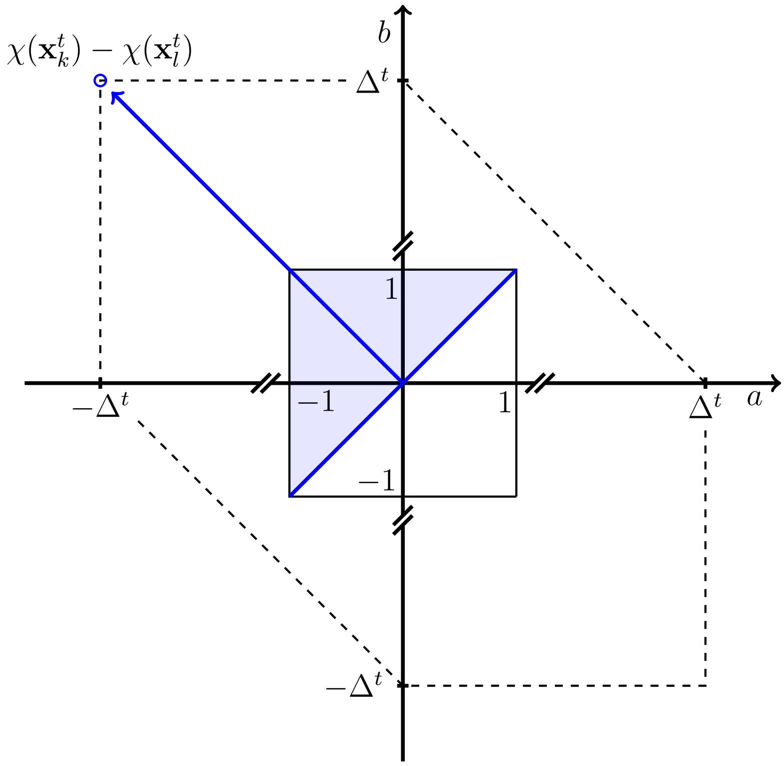
This section establishes that the value profiles of consumers who make the same consumption choice satisfy an appropriate partial ordering. We first derive the condition under which consumers with different value profiles choose the same consumption path $\mathbf{x}^t$. Next, we derive the condition that characterizes the value profiles of consumers who make the same period-t consumption choice x^t, which informs the seller's beliefs about the relevant measures of buyers in PBE (the proofs of all results are relegated to the Appendix).

Lemma 1 (Ordering I: paths). *Consider consumers with common history $h^t \in H^t$. If a consumer with value profile v obtains a higher (present discounted) total utility along path $\mathbf{x}_k^t$ than along path $\mathbf{x}_l^t$, with $\chi(\mathbf{x}_k^t) \neq \chi(\mathbf{x}_l^t)$, then so does a consumer with value profile $\tilde{v} \neq v$ such that*

$$(\tilde{v} - v) \cdot (\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t)) \geq 0. \tag{1}$$

The result shows that, for two consumers with different value profiles to have the same preferences over the total utilities obtained along two distinct consumption paths, the relative value profiles and the relative consumption along the two paths must be aligned. That is, it is generally not sufficient for a type to have strictly higher values for both varieties to satisfy the condition; instead, the relative values $(v_a - v_b)$ must be considered. For example, if following path $\mathbf{x}_k^t$ instead of $\mathbf{x}_l^t$ implies obtaining relatively less consumption of a and relatively more consumption of b and type v chooses path $\mathbf{x}_k^t$ over $\mathbf{x}_l^t$, then only types $\tilde{v}$ who do not prefer a relatively more than b compared to type v will make the same choice. However, if path $\mathbf{x}_k^t$ implies obtaining more consumption of a compared to path $\mathbf{x}_l^t$, while the consumption of b is equal along the two paths, then for type $\tilde{v}$ to have the same preference $\tilde{v}_a > v_a$ is sufficient. Hence, restricting the set of admissible consumption paths makes it easier to satisfy the ordering condition, and restricting the setting to a one-variety problem yields the classic truncation results.

Figure 5: Illustration of the ordering condition in Lemma 1



The intuition of Lemma 1 is illustrated in Figure 5, where the solid square shows the possible differences in the value profiles $(\tilde{v} - v)$ and the dashed lines indicate the possible differences in total consumption $(\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t))$ along two consumption paths. For the particular difference in total consumption $(\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t))$ depicted, only consumers with value profiles in the shaded area satisfy condition (1).[12] If b is an inaccessible state, then differences in total consumption yield vectors along the a-axis only, in which case any higher type $\tilde{v}_a > v_a$ will satisfy condition (1) if $\mathbf{x}_k^t$ implies more consumption of a than $\mathbf{x}_l^t$. This is the classic "skimming" result for one variety.

Lemma 2 (Ordering II: period-t choices). *Consider consumers with common history $h^t \in H^t$. In any PBE, if a consumer with value profile v prefers consumption choice $x^t = x$ to $x^t = x'$, $x' \neq x$, then so does a consumer with value profile $\tilde{v} \neq v$ such that*

$$(\tilde{v} - v) \cdot (x - x') + \delta \left[\min_{\mathbf{x}^{t+1} \in \{\mathbf{X}^{t+1} | x^t = x\}} \left\{ (\tilde{v} - v) \cdot \chi(\mathbf{x}^{t+1}) \right\} \right] - \delta \left[\max_{\mathbf{x}^{t+1} \in \{\mathbf{X}^{t+1} | x^t = x'\}} \left\{ (\tilde{v} - v) \cdot \chi(\mathbf{x}^{t+1}) \right\} \right] \geq 0, \tag{2}$$

where $\{\mathbf{X}^{t+1} | x^t\}$ is the set of admissible consumption paths after consumption choice x^t.

Lemma 2 characterizes the partial ordering applicable in PBE in terms of the values resulting from current consumption choices and (future) admissible consumption paths following these choices. Condition (2) nests well-known earlier skimming results for settings that are covered by our analytical framework. To see this, consider the classic setting with a single durable good, say a, and focus on consumers that have not yet purchased at time t. Let x be the purchase of the durable good, whereas x' is the choice of the outside option, and distinguish the following two cases: $\tilde{v}_a \geq v_a$ and $\tilde{v}_a < v_a$. In the first case, the minimum difference in total value after x equals the maximum difference in total value after x' and is given by $(\tilde{v}_a - v_a)\Delta^{t+1}$, so that (2) simplifies to $\tilde{v}_a \geq v_a$. In the second case, the minimum difference in total value after x is unchanged (but negative), while the maximum difference in total value after x' is 0, such that (2) cannot be satisfied. Thus, we obtain the standard condition $\tilde{v}_a \geq v_a$.[13] A similar result

[12] To see this, note that the sorting condition in Lemma 1 describes a dot product and restricts the angle between the two vectors $(\tilde{v} - v)$ and $(\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t))$ to a maximum of 90°.

[13] We can obtain the same result from (1) by noting that purchasing today (path $\mathbf{x}_k^t$) rather than delaying (path $\mathbf{x}_l^t$) immediately implies that $\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t) \geq 0$.

holds in a setting characterized by positive selection as introduced in Tirole (2016) with a single variety a and an absorbing outside option. Consider the set of types that have purchased a in every previous period until time t. Let x denote the purchase of the good a and x' the choice of the outside option. Then, the minimum difference in total value after x is 0 if $\tilde{v}_a \geq v_a$ and $(\tilde{v}_a - v_a)\Delta^{t+1}$ if $\tilde{v}_a < v_a$, while the maximum difference in total value following x' is 0 for $\tilde{v}_a \geq v_a$ and $\tilde{v}_a < v_a$ because the outside option is an absorbing state. The skimming condition (2) then simplifies to $\tilde{v}_a \geq v_a$.[14]

Finally, consider the two durables setting (Nava and Schiraldi, 2019). Let x be the purchase of one of the two varieties, say a. Then x' is either b or o. Our above analysis of the setting with one durable shows that when $x' = o$, skimming is satisfied whenever $\tilde{v}_a \geq v_a$ if the maximum difference in total value after x' is $(\tilde{v}_a - v_a)\Delta^{t+1}$. Instead, if the maximum difference in total value after x$'$ is $(\tilde{v}_b - v_b)\Delta^{t+1}$, then skimming is satisfied whenever $(\tilde{v}_a - v_a)\Delta^{t} \geq \delta\Delta^{t+1}(\tilde{v}_b - v_b)$. When $x' = b$ in turn, the minimum difference in total value after x is $(\tilde{v}_a - v_a)\Delta^{t+1}$ while the maximum difference in total value after x' is $(\tilde{v}_b - v_b)\Delta^{t+1}$, since both varieties are absorbing, and so (2) becomes $\tilde{v}_a - v_a \geq \tilde{v}_b - v_b$. In conjunction, we obtain that skimming is satisfied if $\tilde{v}_a - v_a \geq \max\{0, \tilde{v}_b - v_b\}$.[15]

3.2 Pricing without trading-up opportunities

Suppose there are no trading opportunities for the seller at history $h^t \in H^t$ with associated state x^{t-1}. It is useful to distinguish the following three different cases:

(i) The state x^{t-1} is absorbing.

(ii) The state x^{t-1} is the most-preferred (accessible) state $i \in (a, b)$ for all consumers, and the transition to the outside option is *not* admissible, $(i, o) \notin \Gamma$.

(iii) The state x^{t-1} is the most-preferred (accessible) state $i \in (a, b)$ for all consumers, and the transition to the outside option is admissible, $(i, o) \in \Gamma$.

[14] Again, we can obtain the result from (1) by noting that $\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t) \geq 0$ because the outside option is an absorbing state.

[15] We can obtain the same result from (1) by noting that for the comparison of two paths that feature different varieties in period t, we have $(\tilde{v} - v) \cdot (\chi(\mathbf{x}_k^t) - \chi(\mathbf{x}_l^t)) \geq 0$ iff $\tilde{v}_a - \tilde{v}_b \geq v_a - v_b$.

It is straightforward to see that pricing dynamics are excluded in cases (i) and (ii) by construction. In case (i), consumers either cannot buy any variety $i \in (a, b)$ (if $x^{t-1} = o$) or must make the same consumption choice i in every future period (if $x^{t-1} = i$), with the price p_i set to zero for all future periods by assumption (and the price p_j set arbitrarily). In case (ii), consumers obtain their most-preferred variety i in every future period at price zero by assumption. For case (iii), our first main result establishes that the seller cannot benefit from dynamic pricing, either.

Proposition 1 (No dynamics after h^t). *Consider history $h^t \in H^t$ with $x^{t-1} = i \in (a, b)$, where i is the most-preferred variety of all consumers. Assume that $(i, o) \in \Gamma$, and suppose that all consumers purchase variety i in the seller-optimal one-shot game that yields the profit supremum, $\pi^*(h^t)$. Then, in any PBE no dynamics in paid prices or realized consumption choices emerge along the equilibrium path starting at history h^t.*

Proposition 1 shows that if, at a given history h^t, all buyers are in their most-preferred state and the seller cannot benefit from pricing out buyers in the one-shot game at this history, then the best the seller can do is to let all buyers continue with purchasing their most-preferred variety at a constant price. Thus, dynamics in paid prices or realized consumption choices do not emerge along the equilibrium path following this history. Note that this result only pins down the price of the most-preferred variety i. For the price of variety j, the only requirement is that it must be sufficiently high to ensure that all buyers continue with purchasing variety i. Thus, dynamics in the price p_j^τ may still emerge, and numerous strategies for the seller may constitute a PBE. However, the seller cannot benefit from dynamic pricing.

The result suggests that if the profit-maximizing solution to the static game does not leave any trading-up opportunities to the seller, then no dynamics will emerge in equilibrium because the seller can exhaust all trading-up opportunities right from the start by playing static optimal prices. Our next result shows that this is indeed the case. Specifically, we consider settings in which there are no trading-up opportunities in the static optimum, so that the monopoly profit $\pi(p^m)$ coincides with the supremum of the profit the seller can obtain in the static game conditional on leaving no trading-up opportunities, $\pi(\bar{p})$. We show that, in this case, the maximum profit the seller can obtain in the dynamic game is the repeated monopoly profit, and the seller can obtain this payoff irrespective of commitment ability. That is, the seller faces no commitment problem if there are no trading-up opportunities in the static monopoly outcome, and the equilibrium is *essentially unique* in the sense that the seller will obtain this profit in any PBE.

Borrowing terminology from Board and Pycia (2014), we say that the seller and consumers adopt "monopoly strategies" if, in every period t, (i) the seller plays p_i^m for every variety i that satisfies $(i, o) \in \Gamma$, and $p_i^m \Delta^t$ for every variety that satisfies $(i, o) \notin \Gamma$ in every state $x \neq i$ (and zero otherwise), and (ii) consumers behave as if they were making optimal choices in the static game facing prices p^m. We can then state the following result.

Proposition 2 (Repeated static monopoly). *Suppose there are no trading-up opportunities in the static monopoly outcome, that is, $\pi(p^m) = \pi(\bar{p})$. Then,*

(i) the seller can do no better than obtain the repeated monopoly profit $\pi(p^m)$ over all periods $t = 0, ..., T$, that is, $\Pi \leq \pi(p^m)\Delta$.

(ii) there exists a PBE in which the seller and consumers adopt monopoly strategies in every period $t = 0, ..., T$.

(iii) the PBE is essentially unique, and thus the seller is guaranteed to obtain the commitment profit irrespective of commitment ability.

Proposition 2 shows that the emergence of pricing dynamics crucially depends on the existence of trading-up opportunities in the static monopoly outcome. A profit-maximizing seller engages in dynamic pricing only if doing so allows her to trade up consumers (buyers or non-buyers) to more valuable consumption options over the course of the game. Therefore, if the profit-maximizing solution in the static game leaves no trading-up opportunities, $\pi(p^m) = \pi(\bar{p})$, then the seller can simply repeat the static monopoly solution and obtain the commitment profit irrespective of commitment ability, since the monopoly strategies of the seller and consumers form a PBE. Moreover, the seller is guaranteed to obtain the maximum profit despite being unable to commit as the described PBE is essentially unique.

The result implies that it suffices to know the profit-maximizing solution of the static game to determine the outcome of the repeated game in settings with $\pi(p^m) = \pi(\bar{p})$. The following corollary provides an explicit characterization of such settings. Intuitively, there are two classes of settings in which $\pi(p^m) = \pi(\bar{p})$: settings that exclude trading-up opportunities for arbitrary price profiles (including static monopoly prices), and settings where the distribution of value profiles is such that static monopoly prices happen to leave no trading-up opportunities.

Corollary 1 (**Settings without trading-up**). *Let $(\bar{x}, \Gamma, \mathcal{F})$ represent the setting under study. Then, there are no trading-up opportunities for the seller*

(i) for arbitrary price profiles p, if the initial state $\bar{x}$ is absorbing.

(ii) for arbitrary price profiles p, if the initial state $\bar{x}$ is non-absorbing and the (weakly) most-preferred state for all consumers, and all other accessible states are absorbing (positive selection).

(iii) for any price profile p such that the lowest-value buyer obtains a strictly positive utility in at least one of the accessible states, if the initial state $\bar{x}$ is non-absorbing and the (weakly) least-preferred state for all buyers, and all accessible states are absorbing (Board and Pycia, 2014).

(iv) for arbitrary price profiles p, if the initial state $\bar{x}$ is non-absorbing, all buyers have the same preference ranking over all accessible states, and only transitions from a preferred to a (weakly) less-preferred state are admissible (trading down).

Otherwise, there exist trading-up opportunities at static monopoly prices, unless $\mathcal{F}$ is such that $\pi(p^m) = \pi(\bar{p})$.

Corollary 1 characterizes the settings in which Proposition 2 applies. Case (i) is trivial in the sense that no transition out of the initial state is admissible. Case (ii) describes a setting characterized by positive selection where the initial state is the most-preferred state for all consumers who can transition to less-preferred absorbing states only. Tirole (2016) provides an in-depth analysis of such a setting with a single non-absorbing variety as the initial state and the outside option as an absorbing state.[16] Corollary 1 shows that we can extend such a setting to allow for a second variety while ensuring that $p^m = \bar{p}$ continues to apply by requiring that the second variety is less-preferred and absorbing. This arguably is the essence

[16]See Figure 4b for an illustration of positive selection with a single variety a.

of positive selection: all consumers start in the most-preferred state and can only transition to less-preferred states in which they are "captured", that is, they can only trade down but never back up. With a single variety, this is ensured if the outside option is absorbing. Case (iii) describes a setting in which the initial state is the least-preferred state for all consumers who can transition to more-preferred absorbing states and obtain a strictly positive utility in at least one of them. Board and Pycia (2014) provide a detailed analysis of such a setting where the initial state is the non-absorbing outside option, and the seller offers a single absorbing variety, but consumers can also choose a second absorbing outside option with a strictly positive value for all buyers. We can embed the basic characteristics of such a setting into our framework by restricting the price for one durable variety to be strictly below the lowest value in the support. Case (iv) describes settings in which the initial state is allowed to be any of the three states, but the admissible transitions and value profiles of consumers are such that they may only ever trade down.

In any other setting, static monopoly prices will generally leave trading-up opportunities in the static game. For example, consider a mixed setting with rental variety a and durable variety b, and assume that the initial state is the non-absorbing outside option (see Figure 4c). For a price profile to leave no trading-up opportunities, we must have that the market clears and that all types allocating themselves to the non-absorbing variety prefer it to the absorbing one. Thus, we need to check if $\pi(p^m)$ happens to satisfy these conditions for the given measure $\mathcal{F}$ and associated support V of consumers.

3.3 Trading-up opportunities and pricing dynamics

Let us now consider settings where the seller has trading-up opportunities at the static monopoly outcome (i.e., $\pi(p^m) \neq \pi(\bar{p})$). Examples include the classic setting with a single durable good, the two durables settings, the two rentals settings, and the mixed setting.

Our next result shows that a profit-maximizing seller will engage in dynamic pricing after any history at which trading-up opportunities exist by repeatedly lowering prices until all trading-up opportunities are exhausted. We further characterize the pricing dynamics and provide conditions under which they are played out in finite time.

Proposition 3 (Pricing dynamics). *In any PBE,*

(i) for any history h^t at which there exist trading-up opportunities, the seller trades up a positive measure of types along the equilibrium path.

(ii) the seller will never set a price for a variety i below $\bar{p}_i$ at any history h^t at which the transition to state i is admissible.

(iii) the seller's (present discounted) profit satisfies $\Pi \geq \pi(\bar{p})\Delta$.

(iv) all trading-up opportunities are exhausted in finite time $t \leq T$ if the minimal value of at least one variety and $\pi(\bar{p})$ are strictly positive, and T is sufficiently large.

Proposition 3 demonstrates that trading-up opportunities are the driving force behind pricing dynamics. For a seller who faces trading-up opportunities and lacks commitment ability, it is strictly profit-maximizing to trade up (some) consumers to a higher-valued consumption option and thereby extract a larger surplus from these consumers. However, in order to induce consumers to trade up, the seller must lower the price relative to the price previously offered. Thus, as the game progresses, the seller is lowering prices to trade up more and more consumers. Since any price profile $p \in \Omega$ leaves no trading-up opportunities, neither in the static nor in the dynamic game, the seller will not want to set prices below $\bar{p}$. Hence, the dynamics come to an end at prices $\bar{p}$, provided that transitions to the respective consumption options are admissible.[17] This implies that the seller's profit in the absence of commitment is bounded below at $\pi(\bar{p})\Delta$, which may be strictly positive (depending on the setting under study). The time it takes for the price dynamics to play out depends on the setting under study. However, for all trading-up opportunities to be exhausted in finite time, both the minimum value for at least one variety and the optimal static profit that leaves no trading-up opportunities, $\pi(\bar{p})$, must be strictly positive, and the number of periods of play must be sufficiently large.

[17] The latter qualification is required for price dynamics to end at $\bar{p}$, because otherwise any price is a best-response (including prices below $\bar{p}$)

To understand the intuition for statement (iv), observe that since it is optimal for the seller to engage in trading-up at any history with trading-up opportunities (statement (i)), the seller must decide whether to trade up some or *all* consumers. The more consumers the seller has already traded up in previous periods, the smaller is the extra surplus that can be extracted from the remaining consumers who can still be traded up. Eventually, it no longer pays for the seller to delay the trading up of some lower-value consumers in order to trade up higher-valued consumers earlier on at higher prices, and the seller trades up all remaining consumers instantaneously. But for this to occur in finite time, the seller must be able to strictly increase profit by trading up all consumers at once. If the minimum value of at least one variety is strictly positive—the "gaps" case—and there are sufficiently many periods of play, this is guaranteed as long as $\pi(\bar{p})$ is strictly positive. Otherwise—in the "no gaps case"— the pricing dynamics may continue indefinitely.

4 Implications

In this section, we summarize the implications of our analysis for the characterization of dynamic monopoly pricing. We pay particular attention to the settings with two rentals, two durables, and mixed varieties.

Implication 1. If there are no trading-up opportunities for the seller in the static optimum, there will be no dynamics in the repeated game. Otherwise, Coasian dynamics emerge.

Our analysis highlights that the profit-maximizing prices that leave no trading-up opportunities in the static game play a crucial role in determining the dynamics in equilibrium. In particular, if the static optimum does not leave any trading-up opportunities for the seller, then the equilibrium of the dynamic game is simply a repetition of the static optimum (see Proposition 2). The intuition is straightforward: the exhaustion of all trading-up opportunities can be understood as the depletion of all gains from trade. Hence, if all gains from trade are depleted, there is no incentive to engage in dynamic pricing. There are a number of settings in which this is the case (see Corollary 1).

Implication 2. Prices will eventually equalize for two rentals if the lowest values in the support are equal, while the same is not necessarily true for two durables.

With two rental varieties, prices continue to fall until they reach the lowest values in the support (see Proposition 3). As a consequence, with equal lowest values in the support there is continuous pressure on prices to equalize: whenever prices are different, there is an incentive for consumers to select the lower-priced variety, even if they prefer the more expensive one. Thus, prices must equalize and reach the lower bound of the support in order to exhaust all trading-up opportunities in the static game. In a setting with two durables, in turn, consumers who strategically select their less-preferred variety are "captured" in an absorbing state, so that prices must not necessarily equalize.

Implication 3. All dynamics end in finite time with two durables, but in infinite time with two rentals unless there are two "gaps" in the support.

A marked difference between the two durables and the two rentals settings emerges in the length it takes for the dynamics to play out. The explanation relates to the profit obtained by the seller when leaving no trading-up opportunities in the static game. A seller of two durable varieties can clear the market (and thus leave no trading-up opportunities) with only one price set to zero provided that there exists a positive measure of consumers who strictly prefer the other variety (Nava and Schiraldi, 2019). As a consequence, eventually it is no longer worth delaying the trading-up of all remaining consumers in order to only trade up some higher-valued consumers, and the dynamics come to an end. For a seller of two rental varieties, in turn, this moment never occurs: whenever prices are not equal, some consumers may select their less-preferred variety to benefit from the lower price, and thus trading-up opportunities persist. The exception to this is the two-gaps case, where the seller can obtain a strictly positive profit when leaving no trading-up opportunities in the one-shot game, and these prices can always be played in the repeated game to end dynamics.

Implication 4. With two "gaps" in the support, all dynamics end in finite time in every setting.

The logic that underlies Implication 3 also implies that, in the two-gaps case, dynamics always end in finite time, irrespective of the specific setting. Two gaps in the support ensure that the seller is guaranteed to earn a positive profit in the one-shot game when leaving no trading-up opportunities, and since she can always play these prices to end the dynamics, eventually it is worth doing so instead of continuing a slow decrease of prices over time.

Implication 5. A seller of mixed varieties obtains a positive profit if there are consumers who prefer the rental to the durable variety.

If there are consumers who prefer the rental to the durable variety, the seller can set a low price for the durable variety to clear the market and ensure that only consumers who prefer the rental variety select it in the one-shot game. By doing so, the seller can obtain a positive profit while leaving no trading-up opportunities. And since these prices can always be played in the repeated game to end all dynamics, she will not be forced to further lower prices and can obtain a positive profit. A seller of a single product, in turn, cannot stop herself from lowering prices to trade up consumers and therefore has an incentive to introduce a low-quality durable variety that is less-preferred by all consumers. This extends a key insight from Nava and Schiraldi (2019) for two durables to the case of mixed varieties.

5 Extension: Transitional games

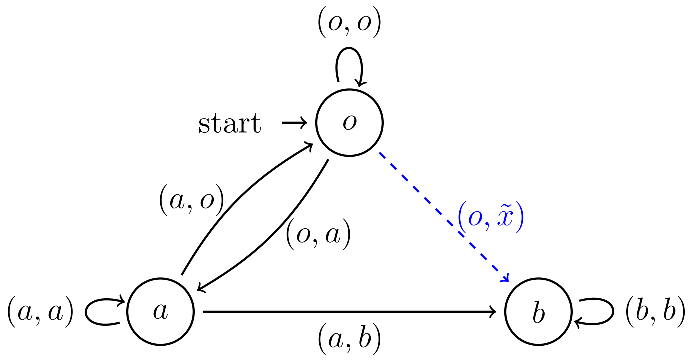
In this section, we sketch how our analysis can be extended to settings in which one of the varieties is only indirectly accessible via the other variety from the initial state $\bar{x} = o$. We call this class of settings "transitional games." Figure 6 provides an example of a transitional game where variety b is absorbing and only indirectly accessible via variety a from the initial state $\bar{x} = o$, with a full support. A possible interpretation is that the seller is offering the baseline version a to all consumers, while the "upgraded" version b is only available to previous buyers of the baseline version. The existence of states that are only indirectly accessible poses a challenge for our analysis, because the consumption options accessible in the static game do not correspond to those in the repeated game, such that there might be no prices that exhaust all trading-up opportunities in the static game.

We now show that our approach of characterizing the dynamic equilibrium by analyzing the associated static game can also be applied to transitional games by introducing a suitably defined "extended static game."

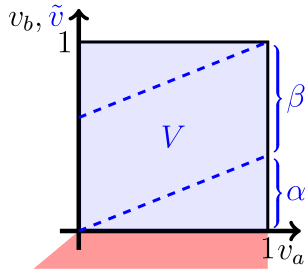
Definition 2 (**Extended static game**). *In the extended static game associated with the transitional game* $(\bar{x}, \Gamma, \mathcal{F})$, *consumers choose among the directly accessible states* $\bar{x}, x' \in X$ *and a mixed state* $\tilde{x}$ *associated with path* $\tilde{\mathbf{x}} = \{x', x'', ..., x''\} \in \mathbf{X}$, *where* $x'' \in X$ *is the indirectly accessible state. The value of the mixed state is* $\tilde{v} = \alpha v_a + \beta v_b$ *for all* $v \in V$, *with* $\alpha = 1/\Delta$ *and* $\beta = (\sum_{t=1}^{T} \delta^t)/\Delta$.

Figure 6: A transitional game

(a) States and transitions



(b) Support



The extended static game is illustrated in Figure 6. In panel (a), the only indirectly accessible state b is replaced by a directly accessible mixed state $\tilde{x}$, and a dashed direct transition $(o, \tilde{x})$ to the mixed state $\tilde{x}$ is added. The mixed state $\tilde{x}$ reflects a one-time deviation from the repeated play of b, which is not possible in the transitional game because state b is not accessible in the first period. Note that the value of the mixed option $\tilde{v}$ is different from v_b for all off-diagonal value profiles in the support. The support of the value profiles $(v_a, \tilde{v})$ in the extended static game follows from a transformation of the original support of $v = (v_a, v_b)$ and lies inside the dashed lines in panel (b).

We denote the price profile associated with the profit supremum that leaves no trading-up opportunities in the extended static game by $\bar{p}^e$. Crucially, setting prices to zero in the extended static game leaves no trading-up opportunities, and hence $\bar{p}^e$ always exists. The next result exploits the definition of the extended static game to show that the repeated profit $\pi^e(p^m)\Delta$ is the equilibrium profit in the transitional game if $\pi^e(\bar{p}^e) = \pi^e(p^m)$.

Proposition 4 (**Transitional game**). *For any transitional game* $(\bar{x} = o, \Gamma, \mathcal{F})$, *there exists an essentially unique PBE in which the seller obtains the repeated* *monopoly profit from the associated extended static game $\pi^e(p^m)$ over all periods $t = 0, ..., T$, if $\pi^e(p^m) = \pi^e(\bar{p}^e)$.*

In conjunction with statement (i) from Proposition 3, which directly applies to transitional games, Proposition 4 implies that our main insights also apply to transitional games, including our approach of checking for trading-up opportunities in the (extended) static game in order to characterize the outcome of the repeated game. However, a subtle difference arises with transitional games: $\pi^e(p^m) = \pi^e(\bar{p})$ no longer excludes that price dynamics emerge in the transitional game, but rather that the only price change that may emerge is a one-time price change that occurs in line with the change in the consumption choice along path $\tilde{\mathbf{x}}$. This one-time price change accounts for the change in available consumption options after the first period and can take the form of a price *increase* for repeat buyers of the same variety.[18]

Figure 6 illustrates a setting with a one-time price increase. Just as in the setting with mixed varieties, the seller can set a price of zero for the mixed option $\tilde{x}$ and a strictly positive price for the rental variety a, thereby ensuring that no trading-up opportunities are left while achieving a positive profit for the given support, $\pi^e(\bar{p}^e) > 0$. But to implement this in the repeated game, the seller must set a price of zero for variety a in the first period, since playing a negative price for variety b in future periods cannot constitute a PBE.

6 Conclusion

This paper employed a unified analytical framework to study a class of dynamic monopoly pricing problems that includes settings with multiple durable, multiple rental, or a mix of varieties. Our analysis demonstrates that the driving force behind pricing dynamics is the seller's incentive to trade up consumers to higher-valued consumption options. We show that the dynamics in equilibrium can be characterized by comparing two solutions of the static game: i) the monopoly outcome, and ii) the optimal outcome for the seller that leaves no trading-up opportunities. If the two solutions coincide, then no dynamics arise in equilib-

[18] A one-time "escalation" of prices in the second period also arises in unit-demand models with a single rental variety (Buehler and Eschenbaum, 2020).

rium, and there exists an essentially unique PBE in which the seller continually plays the monopoly prices. Examples include applications with positive selection (Tirole, 2016) with one or multiple varieties, or multiple durable varieties where one variety provides a strictly positive value (mimicking the setting in Board and Pycia, 2014). Instead, if the monopoly outcome does leave trading-up opportunities, then in equilibrium dynamics arise until all trading-up opportunities are exhausted. These dynamics take the form of Coasian dynamics. We further characterize the conditions under which all dynamics end in finite time, and the final prices at which dynamics end. Examples include settings with one or multiple durable or rental varieties.

We discuss various implications of our analysis. Our findings in particular imply that dynamic monopoly problems can be analyzed by checking for trading-up opportunities in the static optimum. But they also show, for example, that the prices of two rental varieties will eventually equalize if the lowest values in the support are the same, and that a seller of a single rental variety may want to introduce a low-quality, durable variety to avoid being forced down to lower prices. Finally, we consider transitional games, in which one variety is only indirectly accessible and cannot be selected at the start of the game. We show how our analytical approach can be applied in such a setting, and that our key insights translate to transitional games.

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Appendix

Proof of Lemma 1

Since type v obtains a higher total indirect utility along path $\mathbf{x}_k^t$ than along path $\mathbf{x}_l^t$ by assumption, we must have

$$\nu(v, \mathbf{x}_k^t) - \rho(\mathbf{x}_k^t) \geq \nu(v, \mathbf{x}_l^t) - \rho(\mathbf{x}_l^t).$$

Now, consider some type $\tilde{v} \neq v$. Then, we have

$$\begin{aligned}\nu(\tilde{v}, \mathbf{x}_k^t) - \rho(\mathbf{x}_k^t) &= \nu(v, \mathbf{x}_k^t) - \rho(\mathbf{x}_k^t) + \nu(\tilde{v}, \mathbf{x}_k^t) - \nu(v, \mathbf{x}_k^t) \\ &\geq \nu(v, \mathbf{x}_l^t) - \rho(\mathbf{x}_l^t) + \nu(\tilde{v}, \mathbf{x}_k^t) - \nu(v, \mathbf{x}_k^t),\end{aligned}$$

since type v obtains a higher total indirect utility along path $\mathbf{x}_k^t$ than along path $\mathbf{x}_l^t$ by assumption. For type $\tilde{v}$ to obtain a higher total indirect utility along path $\mathbf{x}_k^t$, we must thus have

$$\nu(v, \mathbf{x}_l^t) - \rho(\mathbf{x}_l^t) + \nu(\tilde{v}, \mathbf{x}_k^t) - \nu(v, \mathbf{x}_k^t) \geq \nu(\tilde{v}, \mathbf{x}_l^t) - \rho(\mathbf{x}_l^t),$$

which can be rearranged to yield the result in (1).

Proof of Lemma 2

The line of argument is similar to the proof of Lemma 1. Denoting the continuation valuation of a type v following choice x by $U(v, x)$ (suppressing h^t for brevity) and considering the choices x and x', respectively, we can write

$$\begin{aligned}(\tilde{v} - p) \cdot x + \delta U(\tilde{v}, x) &= (v - p) \cdot x + \delta U(v, x) + (\tilde{v} - v) \cdot x + \delta[U(\tilde{v}, x) - U(v, x)] \\ &\geq (v - p) \cdot x' + \delta U(v, x') + (\tilde{v} - v) \cdot x + \delta[U(\tilde{v}, x) - U(v, x)] \\ &\geq (\tilde{v} - p) \cdot x' + \delta U(\tilde{v}, x').\end{aligned}$$

Rearranging, we have

$$(\tilde{v} - v) \cdot (x - x') + \delta[U(\tilde{v}, x) - U(v, x)] - \delta[U(\tilde{v}, x') - U(v, x')] \geq 0. \qquad (3)$$

Since type v can always mimic the actions of type $\tilde{v}$ (and vice versa) by making the same consumption choices in every future period, the difference in continuation values from period $t+1$ onward must satisfy

$$\min\left\{(\tilde{v}-v)\cdot\sum_{\tau=t+1}^{T}\delta^{\tau-t}\varphi^{\tau}(\tilde{v})\right\}$$
$$\leq U(\tilde{v},x)-U(v,x)\leq \max\left\{(\tilde{v}-v)\cdot\sum_{\tau=t+1}^{T}\delta^{\tau-t}\varphi^{\tau}(\tilde{v})\right\}, \qquad (4)$$

where $\varphi^{\tau}(\tilde{v})=(\varphi_a^{\tau}(\tilde{v}),\varphi_b^{\tau}(\tilde{v}))$ indicates the probabilities associated with type $\tilde{v}$ selecting variety a and variety b, respectively, in period τ. For given types $(\tilde{v},v)$, the min and the max must exclude randomization by consumers, so that (4) becomes

$$\min_{\mathbf{x}^{t+1}\in\{\mathbf{X}^{t+1}|x^t\}}\left\{(\tilde{v}-v)\cdot\chi(\mathbf{x}^{t+1})\right\}$$
$$\leq U(\tilde{v},x^t)-U(v,x^t)\leq \max_{\mathbf{x}^{t+1}\in\{\mathbf{X}^{t+1}|x^t\}}\left\{(\tilde{v}-v)\cdot\chi(\mathbf{x}^{t+1})\right\}.$$

Substituting the boundaries into (3) and reorganizing yields (2).

Proof of Proposition 1

We proceed in three steps. First, we establish that the repeated supremum of the one-shot game profit, $\pi^*(h^t)$, is the highest profit the seller can obtain from buyers with common history h^t if all buyers choose their preferred variety at $\pi^*(h^t)$. Next, we establish that the pure strategies of posting the price $p^*(h^t)$ associated with $\pi^*(h^t)$ in every period and purchasing variety i in every period form a PBE. Finally, we show that whenever any other prices are posted, the seller cannot obtain the highest profit.

We begin by characterizing the seller's present discounted profit at history h^t. Since any strategy profile $\{\sigma,\hat{\sigma}\}$ gives rise to sequences of prices and consumption choices, we can define the seller's present discounted profit at history h^t in terms of the payments made along the admissible consumption paths $\mathbf{x}_k^t\in\mathbf{X}^t$,

$$\Pi(h^t)=\sum_{\mathbf{x}_k^t\in\mathbf{X}^t}\rho(\mathbf{x}_k^t,h^t)\mathcal{F}\left(v\in V(h^t)|\ \mathbf{x}_k^t\right),$$

where the shorthand notation $\mathcal{F}\left(v\in V(h^t)|\ \mathbf{x}_k^t\right)$ indicates the measure of types on path $\mathbf{x}_k^t$, and $\mathbf{X}^t$ is the set of admissible consumption paths at history h^t.

We now derive an auxiliary result that allows us to conveniently rewrite the seller's profit $\Pi(h^t)$. Denote the set of consumers at history h^t who are indifferent between two distinct consumption paths $\mathbf{x}_k^t$ and $\mathbf{x}_l^t$ by

$$V_{k,l}(h^t) \equiv \{v : U(v, \mathbf{x}_k^t, h^t) = U(v, \mathbf{x}_l^t, h^t)\},$$

and the difference in the present discounted values obtained by indifferent consumers with value profile $v \in V_{k,l}(h^t)$ along consumption paths $\mathbf{x}_k^t$ and $\mathbf{x}_l^t$, respectively, by

$$\Delta\nu_{k,l}^t \equiv \nu(v, \mathbf{x}_k^t) - \nu(v, \mathbf{x}_l^t) = \rho(\mathbf{x}_k^t, h^t) - \rho(\mathbf{x}_l^t, h^t).$$

Then, the following result holds.

Lemma 3. *Consider a set of buyers with common history, $V(h^t) \subseteq V$. The seller's present discounted profit at history h^t can be written as*

$$\Pi(h^t) = \rho(\mathbf{x}_0^t, h^t)\mathcal{F}\big(v \in V(h^t)\big) + \sum_{k=1}^{K} \Delta\nu_{k,k-1}^t \mathcal{F}\left(v \in V(h^t) | \ \cup_{j \geq k} \mathbf{x}_j^t\right),$$

where the set of admissible paths $\mathbf{X}^t = \{\mathbf{x}_0^t, ..., \mathbf{x}_K^t\}$ is ordered by the payments such that $\rho(\mathbf{x}_0^t, h^t) \leq \rho(\mathbf{x}_1^t, h^t) \leq ... \leq \rho(\mathbf{x}_K^t, h^t)$, and $\Delta\nu_{k,k-1}^t$ is the difference in the present discounted values obtained by indifferent consumers along consumptions paths $\mathbf{x}_k^t$ and $\mathbf{x}_{k-1}^t$, respectively.

Proof. Let $V_{1,0}(h^t)$ denote the set of value profiles of consumers with common history h^t who are indifferent between $\mathbf{x}_1^t$ and $\mathbf{x}_0^t$. Then for any $v \in V_{1,0}(h^t)$, we have

$$\nu(v, \mathbf{x}_1^t) - \rho(\mathbf{x}_1^t, h^t) = \nu(v, \mathbf{x}_0^t) - \rho(\mathbf{x}_0^t, h^t)$$

or

$$\rho(\mathbf{x}_1^t, h^t) = \rho(\mathbf{x}_0^t, h^t) + \Delta\nu_{1,0}^t$$

by construction. Next, let $V_{2,1}(h^t)$ denote the set of value profiles of consumers with common history h^t who are indifferent between $\mathbf{x}_2^t$ and $\mathbf{x}_1^t$. Then for all $v \in V_{2,1}(h^t)$ we have $\rho(\mathbf{x}_2^t, h^t) = \rho(\mathbf{x}_1^t, h^t) + \Delta\nu_{2,1}^t$, and thus

$$\rho(\mathbf{x}_2^t, h^t) = \rho(\mathbf{x}_0^t, h^t) + \Delta\nu_{1,0}^t + \Delta\nu_{2,1}^t.$$

Iterating this procedure for all consumption paths up to $\mathbf{x}_K^t$ yields

$$\rho(\mathbf{x}_k^t, h^t) = \rho(\mathbf{x}_0^t, h^t) + \sum_{k=1}^{K} \Delta\nu_{k,k-1}^t.$$

Adding up the total payments made by buyers in $V(h^t)$ along the admissible consumption paths, the present discounted profit at history h^t is given by

$$\Pi(h^t) = \rho(\mathbf{x}_0^t, h^t)\mathcal{F}\big(v \in V(h^t)\big) + \sum_{k=1}^{K} \Delta\nu_{k,k-1}^t \mathcal{F}\left(v \in V(h^t) | \ \cup_{j \geq k} \mathbf{x}_k^t\right).$$

□

Now, let σ^* denote the seller's pure strategy of setting the prices $p^*(h^t)$ at every subsequent period $\tau \geq t$ and history h^τ, and suppose that the seller commits to σ^* at history h^t. Then all buyers $v \in V(h^t)$ behave as if they were myopic because they cannot gain from behaving strategically. In addition, let $\hat{\sigma}^*$ denote the buyers' pure strategy of purchasing variety i at prices $p^*(h^t)$. Then, the seller's present discounted profit for the strategy profile $\{\sigma^*, \hat{\sigma}^*\}$ is

$$\Pi(h^t) = p_i^*(h^t)\mathcal{F}(v \in V(h^t))\Delta^t, \tag{5}$$

since all buyers will follow the "always-i path" from t on, $\mathbf{x}_i^t = (i, i, ..., i)$. Lemma 3 implies that this is the maximum profit attainable for the seller. To see this, fix without loss of generality the first path in the order of paths as the always-i-path, $\mathbf{x}_0^t = \mathbf{x}_i^t$. As $v_i > v_j \ \forall v \in V(h^t)$ by assumption, there exists no alternative path $\mathbf{x}_k^t$ with $\Delta\nu_{k,i}^t > 0$. Hence, (5) is the most the seller can obtain at history h^t.

Next, we show that the strategy profile $\{\sigma^*, \hat{\sigma}^*\}$ constitutes a PBE. Suppose that the seller adopts strategy σ^* at h^t and consider the deviation incentives of buyers $v \in V(h^t)$. Buyers who deviate to variety j or the outside option o, respectively, must obtain a lower instantaneous utility if all buyers $v \in V(h^t)$ purchase variety i at prices $p^*(h^t)$ in the one-shot game at history h^t. This instantaneous loss in utility cannot be compensated in the future, because the highest utility any type $v \in V(h^t)$ can obtain from period $\tau \geq t+1$ onward is $(v_i - p_i^*(h^t))\Delta^\tau$. And since the seller obtains the maximum attainable profit by playing σ^* when buyers play strategy $\hat{\sigma}^*$, the strategy profile $\{\sigma^*, \hat{\sigma}^*\}$ is a PBE.

Finally, to prove that the only PBE is one in which $p_i^\tau = p_i^*(h^t)$ and $x^\tau = i$ for all buyers, note that any strategy that results in sequences of prices that ensure all types purchase variety i in every period such that the sum of prices is equal to $p_i^*(h^t)\Delta^t$ is a candidate PBE. In addition, consider that if all types play $x = i$ in the one-shot game at prices $p^*(h^t)$, then it must be that $p_i^*(h^t)$ is the highest price at which all types with a positive density in the support $V(h^t)$ play $x = i$, as otherwise the seller would leave rent on the table. Then it follows that if the seller plays a strategy that results in a price $p_i^\tau > p_i^*(h^t)$, then some types $v_i < p_i^\tau$ must make an instantaneous loss if they accept. As the seller will leave no rent to the lowest type in any future period at any history, this loss can never be recouped. Thus, some types will not purchase at price $p_i^\tau > p_i^*(h^t)$, which must result in a lower profit by Lemma 3. Similarly, any price $p_i^\tau < p_i^*(h^t)$ must be strictly sub-optimal, as the seller will obtain a smaller profit at time τ that can only be recouped by setting a $p_i^\tau > p_i^*(h^t)$ in a future period, at which point some types will not accept. Thus, playing a strategy that results in constant prices at $p_i^*(h^t)$ and sufficiently high prices for $p_j^\tau(h^t)$ to ensure all types play i at every $\tau \geq t$ is the only PBE.

Proof of Proposition 2

(i) First, consider the case of two varieties a, b that satisfy $(a, o), (b, o) \in \Gamma$. Let σ^m be the seller's (pure) monopoly strategy that implements fixed prices $p^m = (p_a^m, p_b^m)$ at every $t \geq 0$ and every history h^t. If the seller commits to σ^m at $t = 0$, we know that all buyers behave as if they were myopic, and the seller's profit is

$$\begin{aligned}\Pi = p_a^m \mathcal{F}\big(v \in V \big| \arg\max_{\mathbf{x}_l \in \mathbf{X}} \{\nu(v, \mathbf{x}_l) - \rho(\mathbf{x}_l)\} = \mathbf{x}_a\big)\Delta \\ + p_b^m \mathcal{F}\big(v \in V \big| \arg\max_{\mathbf{x}_l \in \mathbf{X}} \{\nu(v, \mathbf{x}_l) - \rho(\mathbf{x}_l)\} = \mathbf{x}_b\big)\Delta,\end{aligned}$$

where $\mathbf{x}_a = \{a, a, ..., a\}$ is the always-a path and $\mathbf{x}_b = \{b, b, ..., b\}$ is the always-b path, respectively. Applying Lemma 3, we cannot find a path $\mathbf{x}_k \neq \mathbf{x}_a, \mathbf{x}_b$ such that $\Delta\nu_{k,i} > 0$, with $i \in \{a, b\}$, by the assumption of no trading-up opportunities. Therefore, the seller cannot do better than obtain the repeated monopoly profit $\pi(p^m)$ over all periods $t = 0, ..., T$. Second, note that the seller obtains the same profit Π if, for every variety i with $(i, o) \notin \Gamma$, she instead sets price $p_i^m \Delta^t$ in any state $x \neq i$ (and zero in state i by assumption). Again, buyers behave as if they were myopic, since following a price of $p_i^m \Delta^t$ buyers are guaranteed to obtain zero prices in all future periods by assumption, and there are no trading-up

opportunities by assumption. Hence, the result follows.
(ii) Confirming that monopoly strategies form a PBE is straightforward: if buyers behave as if they were myopic, then setting the monopoly prices p_i^m in every period $t \geq 0$ for all varieties i where $(i, o) \in \Gamma$ and $p_i^m \Delta^t$ in any state $x \neq i$ (and zero in state i by assumption) for all varieties i where $(i, o) \notin \Gamma$, yields the maximum profit; similarly, if the seller repeatedly plays her monopoly strategy, then myopic behavior is optimal.
(iii) Part (i) shows that the highest profit the seller can achieve is the repeated monopoly profit. Part (ii) shows that the seller can obtain this profit in a PBE. Then it follows that the seller will never choose a strategy in PBE that does not deliver the repeated monopoly profit.

Proof of Proposition 3

We prove the four statements in turn.

(i) Fix a PBE. Consider a history h^t on the equilibrium path and denote the state that consumers are in by $x^{t-1} \in X$. Suppose that there exist trading-up opportunities, so that there exists a consumption option $x \in \{a, b\}$ for which some types $v \in V(h^t)$ satisfy $v \cdot x > v \cdot x^{t-1}$ and $(x^{t-1}, x) \in \Gamma$. Denote the set of types that satisfy these conditions by $V^{TU}(h^t) \subseteq V(h^t)$. Let the highest value for variety $i \in \{a, b\}$ for types $v \in V(h^t)$ be $\bar{v}_i$, the lowest value $\underline{v}_i$, and analogously for types $v \in V^{TU}(h^t)$ we have $\bar{v}_i^{TU}$ and $\underline{v}_i^{TU}$. We similarly define $\bar{v}_j, \bar{v}_j^{TU}, \underline{v}_j, \underline{v}_j^{TU}$ for $j \in \{a, b\}, j \neq i$.

In addition, denote the measure of types that are traded up by $\mathcal{F}(v \in V(h^t)|TU)$ and the remaining measure of types that are not traded up by $\mathcal{F}(v \in V(h^t)|NTU)$. By definition, $\mathcal{F}(v \in V(h^t)) = \mathcal{F}(v \in V(h^t)|TU) + \mathcal{F}(v \in V(h^t)|NTU)$. There are four cases to distinguish.

Case 1: $x^{t-1} = j$ and $(j, o) \notin \Gamma$, or $x^{t-1} = o$.
If $x^{t-1} = j$ and $(j, o) \notin \Gamma$, $p_j^t(h^t)$ is set to zero by assumption, and the existence of trading up opportunities implies that (some) types $v \in V^{TU}(h^t)$ will purchase variety i at a strictly positive price $p_i^t(h^t)$, resulting in a profit increase. Similarly, if $x^{t-1} = o$, then inducing (some) types $v \in V^{TU}(h^t)$ to choose $x^t \in \{a, b\}$ constitutes trading up. As the seller earns no profit from types in the outside option, inducing consumers to purchase variety i (or variety j) at a strictly positive price is profit-

increasing.

Case 2: $x^{t-1} = j$ and $(j, o) \in \Gamma$, and $\bar{v}_i^{TU} > \bar{v}_j$.
The equilibrium profit of the seller if she decides not to trade up any buyers, $\hat{\Pi}(h^t)$, satisfies

$$\hat{\Pi}(h^t) < \bar{v}_j \mathcal{F}(v \in V(h^t))\Delta^t, \tag{6}$$

as the seller cannot extract the full surplus of types with a linear price. However, if the seller trades up (some) types $v \in V^{TU}(h^t)$, then the equilibrium profit obtained from trading up, $\Pi^*(h^t)$, satisfies

$$\Pi^*(h^t) \geq v_i^* \mathcal{F}(v \in V(h^t)|TU)\Delta^t, \tag{7}$$

where v_i^* denotes the lowest value v_i of the cutoff types who are indifferent to trading up to i, as the seller can always obtain at least the value of the lowest type in the set. The equilibrium profit obtained from types not traded up, $\Pi^\circ(h^t)$, satisfies

$$\Pi^\circ(h^t) < \bar{v}_j \mathcal{F}(v \in V(h^t)|NTU)\Delta^t, \tag{8}$$

because as before the seller cannot extract the full surplus using a linear price. As $\bar{v}_i^{TU} > \bar{v}_j$ by assumption, there exists a v_i^* that satisfies $\bar{v}_i^{TU} > v_i^* > \bar{v}_j$. Therefore, (6), (7), (8), and $\mathcal{F}(v \in V(h^t)) = \mathcal{F}(v \in V(h^t)|TU) + \mathcal{F}(v \in V(h^t)|NTU)$ together imply that

$$\Pi^*(h^t) + \Pi^\circ(h^t) > \hat{\Pi}(h^t).$$

Case 3: $x^{t-1} = j$ and $(j, o) \in \Gamma$, and $\bar{v}_i^{TU} < \bar{v}_j$; non-absorbing outside option. Suppose the seller does not trade up any types to variety i along the equilibrium path. Then, for any types that play $x^t = o$, we have that Case 1 applies at time $t+1$, and thus trading up is profit-increasing. Specifically, suppose all types $v \in V^{TU}(h^t)$ play $x^t = o$. Since consumers only ever purchase at a price at which they earn a (weakly) positive utility over the course of the game, we know that if the seller never trades up any types $v \in V^{TU}(h^t)$ to i along equilibrium play after $x^t = o$, then since Case 1 applies at any history at which the state is the outside option, it is profit-increasing for the seller to set $p_j^\tau(h^\tau) \leq \bar{v}_j^{TU}$ for some $\tau > t$. But then we can find a $p_i^\tau(h^\tau) > p_j^\tau(h^\tau)$ such that $v_i^*(h^\tau) > \min\{v_j\} \in V(h^t)$, which implies that inducing some types $v \in V^{TU}(h^t)$ to play $x^\tau = i$ is strictly profit-increasing, since the equilibrium profit of the seller when trading up satisfies

$$\Pi^*(h^\tau) \geq v_i^* \mathcal{F}(v \in V(h^\tau)|TU)\Delta^\tau.$$

Now suppose instead (some) types $v \in V^{TU}(h^t)$ play $x^t = j$. Then again, we must have $p_j^\tau(h^\tau) \leq v_j^{TU}(h^\tau)$ for the types $v \in V^{TU}(h^t)$ who are willing to play $x^\tau = j$, such that trading up to variety i is profit-increasing by the above argument. For those types who play $x^\tau = o$, the above argument applies. Thus, trading up must be strictly profit-increasing.

Case 4: $x^{t-1} = j$ and $(j, o) \in \Gamma$, and $\bar{v}_i^{TU} < \bar{v}_j$; absorbing outside option. Suppose that (some) types $v \in V^{TU}(h^t)$ play $x^t = j$. If the seller never trades up any types to variety i, then we know from Case 3 that $p_j^t(h^t) \leq \bar{v}_j^{TU}$, and we can thus find a $p_i^t(h^t) > p_j^t(h^t)$ which ensures that trading up to variety i is strictly profitable. Suppose instead now that all types $v \in V^{TU}(h^t)$ play $x^t = o$. As $x^{t-1} = j$ by assumption, at time $t-1$ we must have had $p_j^{t-1}(h^{t-1}) < p_i^{t-1}(h^{t-1})$ by incentive compatibility, since the indifference condition at $t-1$ is

$$v_i - p_i^{t-1}(h^{t-1}) + \delta U(v, h^{t-1}, x^{t-1} = i) = v_j - p_j^{t-1}(h^{t-1}) + 0, \qquad (9)$$

for types $v \in V^{TU}(h^t)$. As all types $v \in V^{TU}(h^t)$ play $x^t = o$ in period t, we know that $p_j^t(h^t) \geq \bar{v}_j^{TU}(h^t)$ and that $p_j^t(h^t) > p_j^{t-1}(h^{t-1})$. But if it is profit-increasing to induce $x^t = o$ for all types by setting $p_j^t(h^t)$ in period t, the seller could have increased profit by playing $p_j^{t-1}(h^{t-1}) = p_i^{t-1}(h^{t-1})$ in period $t-1$. To see this, note that by setting these prices in $t-1$, the seller assures that no type plays $x^{t-1} = j$, while the profit from types playing $x^{t-1} = i$ at h^{t-1} must be increasing, as the price $p_i^{t-1}(h^{t-1})$ remains unchanged, and the measure of types playing $x^{t-1} = i$ increases. As the seller can always induce these additional types to play $x^t = o$ at time t, the continuation profit is unaffected. Thus, a history where the seller induces $x^t = o$ after $x^{t-1} = j$ for all types cannot arise in equilibrium.

Then in conjunction statement (i) follows.

(ii) Denote by Λ the set of price profiles p that leave no trading-up opportunities for any history h^t in the dynamic game. We will show that $\Omega \setminus \Lambda = \emptyset$ and $\bar{p} \in \Lambda$. Consider the price profile $\bar{p} = (\bar{p}_a, \bar{p}_b)$. Note first, that because $\bar{p} \in \Omega$ by assumption, a price profile $\tilde{p}$ on the diagonal through the type space, with $\eta \geq 0$, satisfies

$$\tilde{p} = (\min\{\bar{p}_a, \bar{p}_b\}, \min\{\bar{p}_a, \bar{p}_b\}) - (\eta, \eta) \implies \tilde{p} \in \Omega, \tag{10}$$

as all types willing to purchase at prices $\tilde{p}$ choose their most-preferred variety, and all types choosing the outside option will also do so at prices $\bar{p}$. Similarly, a price profile $\tilde{\tilde{p}}$ on the (vertical or horizontal) line between $\bar{p}$ and the diagonal, with $\eta \in [0, \max\{\bar{p}_a, \bar{p}_b\} - \min\{\bar{p}_a, \bar{p}_b\}]$, satisfies

$$\tilde{\tilde{p}} = \begin{cases} (\bar{p}_a, \bar{p}_b) - (0, \eta), & \text{if } \bar{p}_b > \bar{p}_a \\ (\bar{p}_a, \bar{p}_b) - (\eta, 0), & \text{if } \bar{p}_b < \bar{p}_a \end{cases} \implies \tilde{\tilde{p}} \in \Omega, \tag{11}$$

as all types purchasing a different variety at prices $\tilde{\tilde{p}}$ than at prices $\bar{p}$ must now choose their most-preferred variety, and all types switching from the outside option to consumption must now choose their most-preferred variety. Second, observe that the price profile $p^\circ = (-\Delta^{t+1}, -\Delta^{t+1})$ is contained in Λ. To see this, recall from the proof of Lemma 2 that types can always mimic each other's behavior (i.e., make the same choices from t onward), so that we have

$$U(\tilde{v}, h^t, x^t) - U(v, h^t, x^t) \leq \max_{i \in \{a,b\}} \{\tilde{v}_i - v_i\} \Delta^{t+1}, \quad v \neq \tilde{v},$$

where U denotes the continuation valuation following choice x^t. Since the maximum value difference satisfies $\max_{i \in \{a,b\}} \{\tilde{v}_i - v_i\} = 1$, all types purchase their most-preferred variety when facing prices p°. In addition, by (10) we also have that $p^\circ \in \Omega$.

Now pick a price profile $\hat{p}$ that satisfies $\hat{p} = p^\circ + (\varepsilon, \varepsilon)$ for some $0 \leq \varepsilon \leq \Delta^{t+1} + \min\{\bar{p}_a, \bar{p}_b\}$. By (10) we know $\hat{p} \in \Omega$. Denote by x° the choice that buyers make in the static game when facing prices p°. By (10) we then have

$$x^\circ \cdot (v - p^\circ) \geq x' \cdot (v - p^\circ), \quad x^\circ \in \{a, b\}, x' \neq x^\circ, \quad \forall\, v \in V, \tag{12}$$

where we know that $x^\circ \in \{a, b\}$ as $p^\circ_a = p^\circ_b < 0$. Since $p^\circ \in \Lambda$, we also have that

$$\begin{aligned} &x^\circ \cdot (v - p^\circ) + \delta U^\circ(v, h^t) \\ &\qquad \geq x' \cdot (v - p^\circ) + \delta U'(v, h^t), \quad x^\circ \in \{a, b\}, x' \neq x^\circ, \quad \forall\, v \in V, \end{aligned} \tag{13}$$

where U° and U' denote the continuation valuations associated with the choices x° and x' respectively, given history h^t. By the definition of p° and (13) it then follows that

$$\delta(U' - U^\circ) \leq (x^\circ - x') \cdot v, \;\; x^\circ \in \{a, b\}, x' \neq x^\circ, \;\; \forall\, v \in V, \tag{14}$$

which also implies that

$$\begin{aligned} &x^\circ \cdot (v - p^\circ - \varepsilon) + \delta U^\circ(v, h^t) \\ &\qquad\qquad \geq x' \cdot (v - p^\circ - \varepsilon) + \delta U'(v, h^t), \qquad x^\circ \in \{a, b\}, x' \neq x^\circ, \;\; \forall\, v \in V. \end{aligned} \tag{15}$$

Thus, for any $0 \leq \varepsilon \leq \Delta^{t+1} + \min\{\bar{p}_a, \bar{p}_b\}$, only types $v < \min\{\bar{p}_a, \bar{p}_b\}$ prefer o to x°, which continues to leave no trading-up opportunities as $\bar{p} \in \Omega$ by assumption, and thus $\hat{p} \in \Lambda$. Hence, for any $\tilde{p}$ that satisfies (10) we have $\tilde{p} \in \Lambda$.

Now fix the price profile $\hat{p} = (\min\{\bar{p}_a, \bar{p}_b\}, \min\{\bar{p}_a, \bar{p}_b\})$. By (11) we have $\hat{p} \in \Omega$, and as shown above we also have $\hat{p} \in \Lambda$. Consider a price profile $p' = \hat{p} + (0, \varepsilon)$ if $\bar{p}_b > \bar{p}_a$ and $p' = \hat{p} + (\varepsilon, 0)$ if $\bar{p}_b < \bar{p}_a$ where $\varepsilon \in (0, \max\{\bar{p}_a, \bar{p}_b\} - \min\{\bar{p}_a, \bar{p}_b\}]$. Then by the same logic as above, for any $\max\{\bar{p}_a, \bar{p}_b\} - \min\{\bar{p}_a, \bar{p}_b\} \geq \varepsilon > 0$, we find that the only types that now prefer the outside option to consumption also prefer the outside option at prices $\bar{p}$ and the only types now preferring the other variety also do so at prices $\bar{p}$. Thus, we find $p' \in \Lambda$ or equally that any $\tilde{p}$ that satisfies (11) satisfies $\tilde{p} \in \Lambda$ and therefore $\bar{p} \in \Lambda$.

Finally, note that we can construct (10) and (11) for any price profile $p \in \Omega$ and thus we find that $\Omega \setminus \Lambda = \emptyset$. Then statement (ii) follows from the definition of $\bar{p}$.

(iii) From (ii), the seller can obtain $\pi(\bar{p})$ for all states x that satisfy $(x, o) \in \Gamma$ in any period t by playing $\bar{p}$ at every history h^t where allowed. By incentive compatibility, if there exist histories h^t with associated state $(x, o) \notin \Gamma$, then playing $\bar{p}_i \Delta^\tau$ at a $\tau < t$ where no such histories exist for any variety i satisfying $(i, o) \notin \Gamma$ and $\bar{p}_i$ for all other varieties and in all future periods will yield $\pi(\bar{p})\Delta^\tau$. The present discounted profit must therefore satisfy $\Pi \geq \pi(\bar{p})\Delta$.

(iv) Consider a history h^t with associated state $x^{t-1} \in X$. Suppose there are trading-up opportunities, so that there exists a consumption option $i \in \{a, b\}$ for which some types $v \in V(h^t)$ satisfy $v \cdot i > v \cdot x^{t-1}$ and $(x^{t-1}, i) \in \Gamma$. Denote the highest and lowest value among these types by $\bar{v}_i^{TU}(h^t)$ and $\underline{v}_i^{TU}(h^t)$, respectively.

Define analogously $\bar{v}_j^{TU}, \underline{v}_j^{TU}$ for $j \in \{a, b\}, j \neq i$, if trading-up opportunities exist for j as well. Denote the set of types that can be traded up by $V^{TU}(h^t) \subseteq V(h^t)$, and the measure of types that can be traded up by $\omega(h^t) = \mathcal{F}\left(v \in V^{TU}(h^t)\right)$.

Let $\bar{v}^{TU}(h^t) = \max\{\bar{v}_i^{TU}(h^t), \bar{v}_j^{TU}(h^t)\}$ and $\underline{v}^{TU}(h^t) = \min\{\underline{v}_i^{TU}(h^t), \underline{v}_j^{TU}(h^t)\}$ denote the highest and lowest value, respectively, of the varieties that consumers at history h^t can be traded up to. Assume without loss of generality that $\underline{v}_i^{TU}(h^t) \leq \underline{v}_j^{TU}(h^t)$ and consider some $\varepsilon(h^t)$ that satisfies

$$\varepsilon(h^t) \geq \bar{v}^{TU}(h^t) - \underline{v}_i^{TU}(h^t).$$

As the seller trades up a positive measure of consumers at any history h^t with trading-up opportunities (see part (i)), by definition of $\bar{v}^{TU}(h^t)$ we have that $\bar{v}^{TU}(h^t) - \underline{v}_i^{TU}(h^t)$ must decrease with the length of a history by Lemma **??**, such that a smaller $\varepsilon(h^t)$ will satisfy the above condition. We now show that for $\varepsilon(h^t)$ small enough, the seller strictly prefers to trade up all types at once if the minimal value of at least one variety and $\pi(\bar{p})$ are strictly positive. To ease notation, we henceforth suppress the conditioning of ω, ε, $\bar{v}^{TU}$ and $\underline{v}_i^{TU}$ on history h^t whenever possible.

As trading up will occur along the equilibrium path for any history with trading-up opportunities (see part (i)), consider t to be the period at which trading-up is profit increasing for the seller for the given history. Let $\Pi^*(h^t)$ denote the equilibrium profit for the seller obtained from trading up only some of the types $v \in V^{TU}(h^t)$. As the seller cannot extract the full surplus with a linear price or trade up the remaining types before time $t+1$, there exists a $\lambda \in (0, 1)$ such that

$$\Pi^*(h^t) < \lambda\omega\bar{v}^{TU}\Delta^t + \delta(1-\lambda)\omega\bar{v}^{TU}\Delta^{t+1}.$$

In addition, let $\bar{\Pi}(h^t)$ denote the seller's equilibrium profit obtained from trading up all buyers. Let $\varphi \in [0, 1]$ be the share of types optimally traded up to j. As the seller can always obtain at least the minimal value of a variety in each period, we have that

$$\bar{\Pi}(h^t) \geq (1-\varphi)\omega\underline{v}_i^{TU}\Delta^t + \varphi\omega\underline{v}_j^{TU}\Delta^t.$$

Using these profits and noting that $\delta\Delta^{t+1} = \Delta^t - 1$, we can write

$$\begin{aligned}
\Pi^*(h^t) - \bar{\Pi}(h^t) &< \lambda\omega\bar{v}^{TU}\Delta^t + \delta(1-\lambda)\omega\bar{v}^{TU}\Delta^{t+1} - (1-\varphi)\omega\underline{v}_i^{TU}\Delta^t - \varphi\omega\underline{v}_j^{TU}\Delta^t \\
&= \left[(\Delta^t - 1 + \lambda)\bar{v}^{TU} - (1-\varphi)\underline{v}_i^{TU}\Delta^t - \varphi\underline{v}_j^{TU}\Delta^t\right]\omega \\
&\leq \left[(\Delta^t - 1 + \lambda)(\varepsilon + \underline{v}_i^{TU}) - (1-\varphi)\underline{v}_i^{TU}\Delta^t - \varphi\underline{v}_j^{TU}\Delta^t\right]\omega \\
&= \left[(\Delta^t - 1 + \lambda)\varepsilon - (1-\lambda)\underline{v}_i^{TU} + \varphi\Delta^t(\underline{v}_i^{TU} - \underline{v}_j^{TU})\right]\omega.
\end{aligned}$$

Therefore, $\bar{\Pi}(h^t) > \Pi^*(h^t)$ whenever

$$\varepsilon(h^t) \leq \frac{(1-\lambda)\underline{v}_i^{TU}(h^t) + \varphi\Delta^t(\underline{v}_j^{TU}(h^t) - \underline{v}_i^{TU}(h^t))}{\Delta^t - 1 + \lambda}.$$

That is, in PBE the seller eventually exhausts all trading-up opportunities at history h^t if $\underline{v}_i^{TU} > 0$ or $\varphi(\underline{v}_j^{TU}(h^t) - \underline{v}_i^{TU}(h^t)) > 0$ and $t \leq T$ is sufficiently large. To conclude the proof, we now show that, if $\pi(\bar{p}) > 0$, then $\varphi > 0$ whenever $\underline{v}_i^{TU}(h^t) = 0$ and $\underline{v}_j^{TU}(h^t) > 0$. Consider that $\pi(\bar{p}) > 0$ implies $\bar{p}_j > 0$ for some $j \in \{a, b\}$. As shown in the proof of (ii), for any history h^t all price profiles $p = (\bar{p}_i, p_j)$ that satisfy $\bar{p}_j > p_j > 0$ ensure that all trading-up opportunities are exhausted and all types behave as if they were myopic, and thus some $v \in V(h^t) \subseteq V$ will consume j, even if $\underline{v}_i^{TU} = \min\{v_i \in V\} = 0$. Hence, $\varphi > 0$ is possible if $\pi(\bar{p}) > 0$. Finally, by the definition of $\bar{\Pi}(h^t)$ we have that $\bar{\Pi}(h^t|\varphi > 0) > \bar{\Pi}(h^t|\varphi = 0)$ if $\underline{v}_i^{TU}(h^t) = 0 < \underline{v}_j^{TU}(h^t)$. Then statement (iv) follows.

Proof of Proposition 4

We first show that, for any strategy profile $\{\sigma, \hat{\sigma}\}$ of the repeated game, we can define an associated static game that delivers the same payoffs to all players when multiplied with the number of periods Δ. The extended static game of Definition 2 is a special case. Since any strategy profile gives rise to sequences of prices and consumption choices, the seller's present-discounted total profit can be written as

$$\Pi = \sum_{\mathbf{x}_k \in \mathbf{X}} \rho(\mathbf{x}_k)\mathcal{F}\left(v \in V|\ \mathbf{x}_k\right),$$

where the shorthand notation $\mathcal{F}(v \in V|\ \mathbf{x}_k)$ denotes the measure of types on path $\mathbf{x}_k \in \mathbf{X}$. Now let $\bar{\rho}(\mathbf{x}_k)$ denote the per-period profit that, if it was received in every period, would give the seller the same (present-discounted) total profit as consumption path $\mathbf{x}_k$,

$$\bar{\rho}(\mathbf{x}_k) = \frac{\rho(\mathbf{x}_k)}{\Delta}.$$

Similarly, let $\bar{\nu}(v, \mathbf{x}_k)$ be the per-period consumption value that, if it was received in every period, would give a consumer of type v the same (present-discounted) total value as that obtained along path $\mathbf{x}_k$,

$$\bar{\nu}(v, \mathbf{x}_k) = \frac{\nu(v, \mathbf{x}_k)}{\Delta}.$$

Then, the seller's present-discounted profit becomes

$$\Pi = \Delta \sum_{\mathbf{x}_k \in \mathbf{X}} \bar{\rho}(\mathbf{x}_k) \mathcal{F}\left(v \in V | \; \mathbf{x}_k\right),$$

and the payoff of a type-v consumer along path $\mathbf{x}_k$ becomes $\Delta(\bar{\nu}(v, \mathbf{x}_k) - \bar{\rho}(\mathbf{x}_k))$. Note that, in the static game associated with strategy profile $\{\sigma, \hat{\sigma}\}$, the seller obtains the per-period payoffs obtained along all paths $\mathbf{x}_k$ multiplied by the (present discounted) number of periods Δ. Each path $\mathbf{x}_k \neq \mathbf{x}_o, \mathbf{x}_a, \mathbf{x}_b$ represents a mixed consumption option.

Now consider the transitional game. Let x' denote the directly accessible variety and suppose that the profit supremum in the associated extended static game $\pi^e(p^m)$ is such that all consumers choose either $\bar{x}, x'$, or $\tilde{x}$, with associated prices $p_{x'}, p_{\tilde{x}}$. Then, profit in the extended static game is

$$\begin{aligned} \pi^e(p^m) = {} & p_{x'} \mathcal{F}\left(v \in V | \; v_{x'} - p_{x'} \geq \max\{\alpha v_a + \beta v_b - p_{\tilde{x}}, 0\}\right) \\ & + p_{\tilde{x}} \mathcal{F}\left(v \in V | \; \alpha v_a + \beta v_b - p_{\tilde{x}} \geq \max\{v_i - p_i, 0\}\right). \end{aligned}$$

By construction, profit in the repeated game must then equally be maximized by inducing consumers to only follow paths $\mathbf{x}_o, \mathbf{x}_i$, and $\tilde{\mathbf{x}}$. Additionally, assume that the allocation induced in $\pi^e(p^m)$ leaves no trading-up opportunities, $\pi^e(p^m) = \pi^e(\bar{p}^e)$. Then $\Delta \pi^e(\bar{p}^e)$ is the maximum the seller can obtain in the transitional game.

We now show that there always exist prices that ensure the seller obtains $\pi^e(\bar{p}^e)\Delta$ in the transitional game. Denote by $\tilde{p}$ the tuple of prices $(p^0_{x'}, p^1_{x'}, p_{x''})$ that, if played repeatedly in the associated periods ($p^0_{x'}$ in period $t = 0$, and

$p^1_{x'}, p_{x''}$ from period $t = 1$ onward) yield the same payoff to the seller. Denote this strategy for the seller by $\tilde{\sigma}$. These prices must satisfy the following two restrictions

$$p^0_{x'} + (\Delta - 1)p^1_{x'} = \bar{p}^e_{x'}\Delta,$$
$$p^0_{x'} + (\Delta - 1)p_{x''} = \bar{p}^e_{\tilde{x}}\Delta.$$

By assumption, as $\bar{p}^e \in \Omega^e$, it must be that no types allocate themselves to the outside option $\bar{x} = o$ when facing prices $\bar{p}^e$, and thus in the repeated game no types follow path $\mathbf{x}_o$. Consider the indifference condition between x' and $\tilde{x}$ in the extended static game,

$$v_{x'} - \bar{p}^e_{x'} \geq \tilde{v} - \bar{p}^e_{\tilde{x}}$$

Substituting the conditions on prices $\tilde{p}$, we obtain

$$v_{x'} - p^1_{x'} \geq v_{x''} - p^1_{x''}. \tag{16}$$

Thus, prices $\tilde{p}$ must ensure that the following two conditions hold

$$v_{x'} - p^0_{x'} + \delta U(v, x', t = 0) \geq 0 \ \forall v \in V, \tag{17}$$
$$v_{x'} - p^1_{x'} + \delta U(v, x', t = 1) \geq v_{x''} - p^1_{x''} + \delta U(v, x'', t = 1) \ \forall v \in V, \tag{18}$$

where $U(v, x', t = 0)$ denotes the continuation valuation associated with consumption choice x' at $t = 0$.

Consider that if $\bar{p}^e \in \Omega^e$, then (16) must induce an allocation at which all types $v_{x''} > x_{x'}$ play x''. Such prices by definition also leave no trading-up opportunities at the one-shot game at $t = 1$ in the transitional game at state x'. Thus, we can follow the proof of (ii) in Proposition 3 to find that playing $(p^1_{x'}, p^1_{x''})$ induces the same allocation as in the one-shot game.

It only remains to check that playing $\tilde{\sigma}$ at $t = 0$ also ensures that (17) holds. It is straightforward that $U(v, x', t = 0) \geq 0 \ \forall v \in V$ by construction. We therefore find that playing $p^0_{x'} \leq \underline{v}_{x'}$ ensures that (17) is satisfied, where $\underline{v}_{x'} = \min\{v_{x'} \in V\}$. Thus, the seller can play $\tilde{\sigma}$ and all types will allocate themselves as they do in $\pi^e(\bar{p}^e)$, while prices satisfy the necessary restrictions to ensure the sellers profit is $\Delta\pi^e(\bar{p}^e)$.